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ABSTRACT

For the purpose of promoting international cooperation in mathematics, physics, and chemistry teaching research, this first yearbook of the Finnish Association of Mathematics and Science Education Research contains articles of recent research on mathematics teaching. The contents consist of five articles in English, two in German, and one thesis summary. Topics addressed include: (1) the position of mathematics in the school system; (2) mathematics curriculum reform in Scandinavia; (3) longitudinal study of the development of mathematical thought processes and strategies; (4) research on cognitive styles; (5) multiplication skills among German students; (6) mathematical knowledge of Finnish teacher trainees; and (7) application of logistic test models to mathematics tests. Biographic notes of the contributing authors are also included. (ML)

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REPORTS FROM THE INSTITUTE FOR EDUCATIONAL RESEARCH

JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ

MATHEMATICS EDUCATION RESEARCH IN FINLAND
YEARBOOK 1983

Edited by Pekka Kupari

REPORTS FROM THE FINNISH ASSOCIATION OF MATHEMATICS AND
SCIENCE EDUCATION RESEARCH

4

THE FINNISH ASSOCIATION OF MATHEMATIC AND SCIENCE EDUCATION RESEARCH

MATHEMATICS EDUCATION RESEARCH IN FINLAND
YEARBOOK 1983

Edited by Pekka Kupari

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PREFACE

The following reasons gave rise to increasing interest in the area of research into mathematical subjects (mathematics, physics and chemistry) in Finland in the 1970's:

1) The curricula of the mentioned subjects in connection with the school reform were largely debated and development work required a lot of research. For this purpose international comparisons between teaching and learning outcomes (the IEA projects) have been carried out at the Institute for Educational Research, University of Jyväskylä as well as national follow-up on the implementation of the curricula.

2) The structure of mathematical subjects teaching has undergone a great amount of "modernization", which also has required didactic research within the teacher training colleges.

3) Training of comprehensive and upper secondary school teachers became the responsibility of our universities, which resulted in the appointment of new associate professors in mathematics, physics and chemistry didactics. In the meantime students took up research in the area at the stage of their specialization studies. Thus a general interest in research activity has increased.

A need for contacts between researchers is growing while research activity rouses interest. Teacher trainers as well as teachers have become more interested than was the case earlier in research-based information, which has naturally made contacts with those applying the results necessary. The first research seminar on mathematics teaching was arranged at the University of Jyväskylä in February 1982. Its theme was "Cognitive Processes and Mathematics Teaching". The seminar report has been published in the report series of the Institute for Educational Research. A corresponding seminar for physics and chemistry teachers was held in Helsinki in November 1982, and the report has been published by the National Board of General Education, Helsinki.

The second seminar on mathematics teaching was arranged at the University of Turku in August 1983. The Finnish Association of Mathematics Education Research was founded during the seminar. The name was, however, changed some time later to its present form, which stands "The Finnish Association of Mathematics and Science Education Research".

At the organization meeting of the association held in Turku on August 30, 1984 fourty-eight members joined the association. The following functionaries were elected: associate prof. Paavo Malinen, chairman; researcher Pekka Kupari, secretary; assoc.prof. Jarkko Leino, lecturers Paula Paavola and Erkki Pehkonen as members of the board.

The first task of the association was to make plans for a yearbook on research into mathematics teaching. It was not, however, necessary to start the work from basics, because an editing committee for such a yearbook had already in May 1983 been set up on the auspices of the Institute for Educational Research and the National Mathematics Committee. At present the yearbook has been published and it is available for orientation and reading. The articles and reviews are mainly an outcome of the Turku research seminar, some of the contents represents other recent research. Our aim is

to continue in the future as well to present mathematics, physics and chemistry teaching research and seminar papers in order to promote international cooperation in the field.

The Finnish Association of Mathematics and Science Education Research expresses its deepest gratitude to Pekka Kupari and the entire editing board for making this yearbook possible.

On behalf of the Finnish Association
of Mathematics and Science Education

Paavo Malinen
Chairman

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INTRODUCTION

The Finnish educational system has undergone great changes in early 1980's. Since 1982 a new form of secondary education has been implemented and essential changes have taken place in both vocational and upper secondary education. The 1983 Law on the comprehensive school brings forth a change, which means that setting in our comprehensive schools is abandoned from fall 1986 and that the time resource quota-system will be applied in order to make differentiation within teaching groups easier as well as for the purpose of ascertaining eligibility for further studies. The changes have naturally affected mathematics curricula and e.g. the new syllabus for comprehensive school mathematics teaching was approved in fall 1982.

The articles in the first yearbook have no common theme, but they cover the core of various research endeavours and presentations published in 1983. In all the yearbook contains five articles in English, one thesis summary and two articles in German; the other one was written by a German lecturer, who was present at the Turku research seminar.

The first article throws light on the position of mathematics in the present school system. It deals with changes that have taken place in school-mathematics teaching as well as their causes. Problematic content areas, their universality with regard to difficulties and desirable developments in the field are discussed.

The second article describes the reform of mathematics curricula in Scandinavia in the 1960's and 1970's. On one hand the article discuss changes in curricula and the causes for change and on the other hand the effects of changes on pupils' learning outcome.

The third article contains a very thorough description of an experimental study on the interrelationships between cognitive and affective factors which underlie and influence the arithmetical and measurement performance of first-graders in the comprehensive school. The investigation is based on a longitudinal study of the development of mathematical thought processes and strategies begun in autumn 1982.

The two following articles deal with research into cognitive styles, which research field has during the last years attracted a lot of interest. The first of the two articles describes how school achievements are connected with the cognitive profile and how the results of profile studies could be applied to school instruction. The second article deals more specifically with the significance of two cognitive styles in mathematical problem solving.

The first German article presents studies on the multiplication skills among German pupils. The effects of short-term, repeated practise periods on 3-graders' learning of multiplication tables are discussed in the first half of the article. Later the multiplication skills of 5-10-graders attending two different school types, ie. Hauptschule and Realschule are analyzed.

The second German article elucidates the mathematical knowledge of Finnish classteacher trainees at the beginning of training and the said knowledge is compared with the so-called minimum starting point knowledge, which is also presented in the article.

The yearbook ends with a summary of a doctoral thesis approved in 1983. The area of the thesis is how to apply logistic test models to mathematics tests in order to analyze learning difficulties.

QUO VADIS, SCHOOL MATHEMATICS?

Hannu Korhonen
Pekka Kupari

1. Continuous change

It was the introduction of set theory which made school mathematics a highly actual topic at the turn of the sixties and seventies. Teachers and parents hurried tc train themselves to be able to teach new mathematics and to follow their children's progress. Today, when over twenty years have passed since the beginning of the reform, it is possible to evaluate the results and, at the same time, prepare for a new change, for which at least as much preparatory work has been done by UNESCO as was done during the introduction of set theory.

Looking from an international perspective, mathematics teaching has changed most significantly in two respects during the past twenty years. Firstly, while education has reached an ever-growing number of school-aged children, conformed learning objectives have been established for this increasingly heterogenous group of pupils (Morris, 1981, V 113). Secondly, new topics are continously being introduced in school mathematics at the same time as the importance of the traditional "arithmetics" as a basic skill of a citizen is decreasing along the progress of automation. The pressure for the reform of learning contents results to a greater extent from the change and expansion of mathematics itself than by the development needs of pupils and the demands of society. Change is apparently becoming a permanent state because the information collected by

UNESCO (Morris, 1981, 117) states that in many countries continuous curriculum development is considered of greater value than extensive changes taking place only seldom.

Mathematics has always been a highly esteemed school subject. It has been considered important both for the learner's intellectual development and, on the other hand, for its great practical value in everyday life. Oddly enough these aspects appear in current discussions. A UNESCO report published some years ago dealing with the goals of mathematics education (Morris 1981, V) confirms that mathematics has permeated our whole society. Besides those who need mathematics in their profession, also ordinary people are assumed to understand mathematics in their daily life. It is true that statistics, development prognoses, economical statements of the situation and new scientific and technological inventions actually require more understanding than computing. Therefore mathematics teaching cannot and must not comprise only of practices of computational skills based on memory and automation and techniques for solving stereo-typed problems according to a pattern. In view of working life as well as the growing free time, the major contributions of mathematics education to the growing youth obviously include creative, structured thinking abilities and non-stereo-typed problem-solving skills.

Although it is a generally accepted fact that mathematics education must be provided for all because of its social importance and that differences in learners' intelligence and in their learning styles cause enormous difficulties in the arrangement of common instruction and the evaluation of results, people are reluctant as yet to allocate sufficient resources for mathematics education in the West. As early as the mid-seventies it was estimated in the United States (Krygowska 1979, 32) that interest in mathematics education, as well as its financial support, had noticeably decreased since the enthusiasm of the early years of new mathematics. The importance of a nation's brightest mathematical talent for the development of society - regardless of the mathematical skills of the whole

people - is also generally recognized. It is obviously for this reason that the societies which follow the planned economy system pay particular attention to the special education of mathematically gifted pupils.

2. Some research results

Removal of setting from the comprehensive school and the introduction of the course-based upper secondary school signify so great a reform of teaching arrangements that such a reform has not been seen in this country since the time when secondary schools became widespread. In view of mathematics teaching, it is positive that during the past ten years many international and national studies have yielded more research-based information especially on the learning of 13 to 15-year-old pupils on the upper level of the comprehensive school, than all the earlier information put together. Internationally one of the most important projects is the Second International (IEA) Study of Achievement in Mathematics which was organized in Finland in early 1981. A preliminary report has been published already a couple of years ago (Kangasniemi 1982). In 1979 the Institute for Educational Research (KTL) at the University of Jyväskylä conducted an extensive curriculum evaluation project in several school subjects. This research was commissioned by the National Board of General Education and is called the First National Assessment of Teaching in the comperehensive School. The survey of mathematics teaching (Kupari 1980) included grades 4,6 and 9 of the comprehensive school. Approximately 4300 Finnish-speaking pupils from 140 schools and about 3200 Swedish-speaking pupils from 190 schools participated in the study.

Similar research studies related to achievements in mathematics have among others been conducted in the late seventies in Great Britain and the U.S. The most comprehensive American project was the investigation carried out by the NAEP in 1977-78, in which more than 60 000 9 to 17-years-old pupils took part

(NAEP 1979). One of the most significant British studies for the assessment of pupil's developmental stage and their achievements has been a research program (Concepts in Secondary Mathematics and Science) which explored 11 to 16-year-old pupils' understanding of mathematics (Hart 1981). These, and a great number of other investigations which have been carried out around the same period of time, have yielded surprisingly consistent results. In different parts of the world learning outcomes show the same kind of variation and pupils use similar strategies in solving problems. When comparisons are made between the mathematical skills of present-day pupils and earlier age groups, the focus is usually on such fundamental operations of arithmetic which earlier were taught to everyone in the elementary school. From an international viewpoint among the most difficult topics in arithmetic are fractions, which even all adults do not fluently master. In the above-mentioned investigations it was found that the most common error in the addition of fractions was such that pupils first added the numerators and then the denominators. For example nearly one fifth of Finnish fourth-graders in the comprehensive school arrived at the result $\frac{3}{8}$ when they has to solve the following problem: $\frac{2}{4} + \frac{1}{4}$ (Kupari 1981a, 58). Even a somewhat greater number of British school children of the same age made exactly the same error (Foxman et. al. 1980, 46). Typically enough, even 15-year-old British school children did not succeed any better in solving the same problem. This error was made in addition to fractions with no common denominator (e.g. $\frac{2}{5} + \frac{3}{5}$) by a good third of 13-year-old pupils both in Finland, Great Britain and the U.S. (cf. Kupari 1981b, 57; Hart 1981, 75; NAEP 1979, 32). The occurence of the erroneous performance was thus nearly as common in the different countries, and moreover it was astonishingly great.

Why then is this the case? According to both American and British researchers, many pupils do not understand the concept of fractions, and therefore try to add them up mechanically (e.g. Hart 1981, 76). According to Hart these pupils do not feel confident in the use of fractions and try whenever possible

to apply the rules of whole numbers to operations of fractions. In Kamii's (1982) opinion, these pupils rely on any mechanical methods which they have been taught rather than on their own reasoning ability. Kamii states, as far as these pupils are concerned, mathematics is a huge stock of methods by which answers to problems are found and, for their part, the answers always depend on what method the pupils happen to be using. However, in the teaching of arithmetic the major aim should be the development of thinking processes and abstracting ability and not merely the production of correcet answers.

Another example of the study of children's comprehension of mathematical concepts is the study of problems involving percentages which are part of the "everyday practical mathematics" of all citizens. For example, both in our country and in Britain, approximately on third of the 13-years-olds and around one half of 15 to 16-year-old pupils were capable of solving the following problem type: how much is such and such a percentage of number? (cf. Kupari 1981c, 185-186; Hart 1981, 96). Only slightly better results were obtained in solving the following problem: what percentage is a number of another? Furthermore, less than half of the 15 to 16-year-old pupils were able to solve a discount sale problem which indicated the price and the percentage by which it was reduced, and which then asked for the reduced price. Most children tended to use a faulty rule in solving the problem or concentrated on just one word in the problem, e.g. the word 'reduced'.

Again, the results indicate that many pupils have learned the solution formula for each percentage problem type in connection with the examples presented in the teaching situation, but they have very soon forgotten which formula is applicable to each case. Understanding the concept of percentage and its application to various practical situations has remained obscure. American researchers, among others, believe, that satisfactory results cannot be produced by this kind of mechanical application of computational skills which was most frequently used for solving these problems (NAEP 1979, 33).

3. Understanding

The shortcomings detected in the understanding and application of mathematics evidently result from erroneous priorities in the curriculum and also from the nature of instruction. Researchers feel that for example the 'back-to-basics' movement, which has existed in the U.S. and Great Britain since the mid 1970's, has narrowed the curriculum while stressing that teaching should be confined to the basic arithmetic skills (NAEP 1979, 34). Consequently higher mental processes such as understanding and problem-solving have been given quite insufficient attention. As a result, pupils obtain only a weak conception of the inter-relationships of the topics they have learned. It is even possible that computational skills deteriorate when pupils do not understand concepts that occur in the problems.

Study of learning achievements reveals clearly that mathematics is a very difficult subject for many pupils. Teachers often tend to believe that the pupil who enters the upper level has a sufficient store of knowledge on which the abstract structure of mathematics can be built. Every child has some knowledge, but it is seldom as great as expected. In fact, there are gaps even in such simple skills as the performance of fundamental operations of arithmetic on paper or the use of algorithms (e.g. Hart 1981, 212). Too short a time span is used on teaching them. After they have been taught, their use is illustrated by a few simple examples and then it is assumed that once taught they are remembered. As the above examples of fractions indicated, algorithms are not always remembered or they are sometimes remembered in a way that was never taught. Research results show also that the acquisition of knowledge is usually a very slow process and that learning at a level of understanding improves only slightly as children grow older (Hart 1981, 210). The building-up of mathematical knowledge is a process demanding continuous re-teaching of topics that were thought to be already assimilated by the children. Thus the work can only take place "a brick at a time and not a whole wall".

Therefore, in mathematics teaching, we should proceed from the knowledge base which the pupil himself has (e.g. Kamii 1982, 250-251). Pupils must be encouraged to use their own reasoning ability in dealing with and solving problems. A closer examination of erroneous strategies often leads to the right solution, because knowledge is not renewed by just adding on new bits of information to previous information, but rather by modifying of old thoughts and ideas. For this reason, pupils should be encouraged to explain, discuss and defend their own thoughts and ideas, until they convince others of their correctness and logic or until they themselves become convinced of the superiority of the ideas and solutions of the others. If attention is paid to the development of pupils' strategies in solving problems and if the pupils can be made to trust their own faculties of understanding things, they will be sure to get correct answers and they will want to solve more and more challenging problems. The establishment of an enthusiastic and active spirit and attitude, both in teachers and in pupils, plays a central role in the improvement of the quality of mathematics teaching intended for the whole age cohort, as well as in guiding the gifted pupils to take special interest in mathematics.

4. Contests

In 1980, the National Swedish Board of General Education started a biennial series of public surveys on mathematics education. In connection with the first biennial conference it was declared (Biennal-bulletinen 1980) that the aim was to make Sweden the best country in the world in the field of mathematics by the year 1990 ("världsbäst i matte år 1990"). Thus three times already more than a thousand mathematics teachers from the primary level to the upper secondary school have been gathered to the biennials to hear in what direction mathematics education should be developed and - what is most remarkable - to give an enthusiastic and self-confident report of the way in which the teaching of mathematics has been arranged in "our school"

or "our municipality". The standard of the exhibition departments set up by schools has also clearly improved each time.

A very different type of program for the promotion of mathematics education is represented by the mathematics Olympiads for upper secondary school pupils which take place every summer. These Olympics have been organized 24 times already, but only in recent years they have become an internationally acknowledged affair. The two most recent "games" were organized in Hungary and in Paris. A total of 32 countries with their teams of four members participated in the math Olympics last summer. Finland took the 13th position and left far behind our "dear enemy" Sweden, among others. Success in these Olympics probably indicates less the overall level of mathematics teaching in a given country than the opportunities provided for the "top athletes" and the efforts made for their training. The success of Finland has at least improved along the training of its Olympic team, which was selected on the basis of the national mathematics competition for the upper secondary schools. Continuous success is a question of honour to Finland, because Finland vill organize the mathematics Olympiads in 1985.

If math Olympics represent top athletics, also popular sports events are arranged. In the U.S. approximately 420 000 pupils from nearly 7000 schools participated in 1982 in the Annual High School Mathematics Examination (AHSME) competition. In Finland, a proportionally greater number of pupils participated in the mathematics competiton for comprehensive school 9-graders last autumn. Over 40 000 ninth-grades from more than 500 schools participated - which is almost two thirds of the age group. This comprehensive school competition is not primarily aimed at finding talents but rather at increasing pupils' interest in mathematics by giving them problems to think about, which are connected with practical life. Judging by the enthusiasm shown by the great number of participants, the idea of competition even in the intellectual field is not so strange to pupils as people have often been given to understand in the comprehensive school debate.

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THE REFORM OF MATHEMATICS CURRICULA IN THE NORDIC COUNTRIES IN THE LIGHT OF RESEARCH

Paavo Malinen

1. Introduction

After the first Sputnik in 1957 a great interest in the reform of the teaching of mathematics rose in the USA and Western Europe. As a result of that, the OECD organized a seminar in Royaumont (France) whose report "New Thinking in School Mathematics" served as a basis for the process of reform in Western Europe. In 1960 the Council of the Nordic Countries founded a committee called "Nordiska kommittén för modernisering av matematikundervisning" (Finnish abberviation PMOU), in which Norway, Sweden, Finland and Denmark took part. The committee arranged a great many seminars and school experiments and its final report "Nordisk skolmatematik" (Anon. 1967) formed the foundation for the reform of school mathematics in all the Nordic countries.

The consequences of the above reform process have not been systematically investigated. Nordic co-operation in this area continued in the 1970s, too, but only surveys on curriculum development were presented. First, the main domain of co-operation was harmonization, which aimed at standardizing curricula and later at developing individualized instruction. Plans for harmonization seem to have influenced curriculum development in different countries although it has not been thoroughly investigated, either.

The reform of the teaching of mathematics in the 1960s and 1970s included various measures:

1) introduction of the compulsory 9-year comprehensive school, which canged the structure of the curriculum
2) introduction of new subject matter and making the teaching of mathematics an integral whole from the first grade on, which prompted discussion of the "modenrnization" of mathematics teaching
3) development of educational technology and research into the psychology of learning, which called forth changes in the planning process of teaching.

It has been difficult to study all the above canges separately in different schools and hence, only their combined effects can be seen. For example, the followirg methods are available for research:

1) one can evaluate the influence of various factors on written curricula
2) one can evaluate the implementation of new curricula in terms of goals and prior achievement.

Curriculum reform is a diversified process of innovation whose investigation would require careful monitoring. Since such research has not been done, it is possible to examine the situation only on the basis of investigations made afterwards. This report examines the reform of the curricula for mathematics in the Nordic countries in the 1960s and 1970s and special attention is paid to the changes in the curriculum content and to school achievement attained through new curricula.

2. Changes in the national curricula

The most extensive investigation concerning processes of reform in the Nordic countries is Gunnar Gjone's doctoral thesis (1983) at the University of Oslo. The report, published in

eight parts, deals with the factors influencing curriculum development in Norway. The contribution of Nordic co-operation to the reform of the contents was considerable at the end of the 1960s but a great deal of influence came directly from the USA. The official curriculum reform in the comprehensive school was retarded by the fact that in 1971 the drafting committee presented two alternatives for the subject matter - a traditional and a modern one. Their comparison induced a great deal of discussion and finally a compromise was made and the curriculum became established in the middle of the 1970s.

Gjone's main object of research was the analysis of the factors influencing the process of reform. There was so much disagreement on this matter among mathematicians that they could not provide any definite direction to the reform. One controversy of educational policy was the question of the standardization of the curriculum for grade 9 since it was difficult to define general goals and basics. The final curriculum is the result of co-operation between unofficial organs (teacher association, experts, public discussion in the press) with educational policy planners designing the written form.

The methodology of Gjone's investigation follows the tradition of the near past: it is based on archives and interviews. It reveals how difficult it is to design an elaborate curriculum under controversial pressure and also how decision-making is transferred to educational policy planners. The planning process has been analyzed from many points of view and while doing this Gjone has developed applicable models to design the curriculum.

Matts Håstad's doctoral thesis (Håstad 1978) was published in Sweden in 1978. It dealt with the evaluation of the reform in teaching mathematics during the last twenty years mostly in terms of teaching in technical colleges. The teaching of mathematics in these colleges had met with difficulties because of students' poor basic knowledge and the changes in the curricula for the comperehensive and senior secondary schools were

considered responsible for it. According to the research, following factors were most essential in the curriculum reform:

- The overall number of mathematics lessons in the comprehensive and senior secondary schools had diminished from 3100 to 1900, which means that chances for practising calculations had significiantly declined.
- The syllabi of mathematics had undergone rapid changes in their content, which had disturbed the systematic teaching and learning of talented pupils especially.
- Owing to influential tradition, the changing of teaching has been slow and the period of transition difficult.
- Educational policy planners were interested in "new mathematics" and wanted to standardize instruction with it but teachers were not for innovation, which had in a way been forced from outside.

Håstad concludes his investigation by pointing out that in many respects the reform of mathematics teaching did not attain its objectives in Sweden in the 1960s. With the above conclusion he looks down upon his own contribution, too, as he acted as the general secretary of the PMOU (Nordic Committee for the Modernization of Mathematics Teaching) and actively took part in designing the curriculum of 1969. He was also one of the authors of the series of books called "Hej matematik", which was published at the same time. In his opinion the reason for such a poor final outcome was the fact that the issues influencing curriculum reform were not under control. Consequently, Sweden has emphasized the return to the old arithmetic, "back to basics", which according to Håstad's investigation would not seem to improve the situation.

The structure of Håstad's doctoral thesis is incoherent. The data from the recent past have been quite superficially analyzed and original sources have not been utilized. He presents some theoretical examination on the conditions of mathematics teaching but it is only vaguely related to more general theory-forming

of the conditions of instruction. However, because of its comprehensiveness the investigation gives stimuli to other researchers. Furthermore, he merits reward in examining the application of mathematics teaching from the point of view of teaching in the future.

Many other investigations concerning the teaching of mathematics have been published in the PUMP-projekt in Sweden. The most closely related report to the curriculum reform of mathematics is "Hej läroplan" (Kilborn et al. 1977). Through interviews and committee reports it evaluates factors that actually influenced the curriculum for the teaching of mathematics of 1969. The authors are of the opinion that the curriculum was designed simultaneously with the series of books called "Hej matematik", which also accounts for the name of the report. The text is enlivened by numerous quotations from the interviews. The report is more like a survey than an investigation but as such it demonstrates the effects of accidental and practical factors on curriculum design.

The investigations of the PUMP-project have been taken into consideration in Viggo Kilborn's book (1981) as he examines the question of the position of basic subject matter in the curriculum. The influences of Nordic co-operation can also be seen here; the Norwegian "kjernestoff", the Swedish "baskunskaper" and the Finnish "perusoppiaines" have been coherently interpreted on the basis of reserach. The usage of this notion in curricula has not, however, become established.

In Denmark curriculum reform in mathematics has been gradually initiated on local level since only some recommendations are given on national level. The most visible feature has been development in teacher training where especially Danmarks Lærerhöjskole has taken an active part in training teachers to implement new curricula. Real investigations on curriculum reform have not been available.

In Finland the curricula were gradually changed at the beginning of the 1970s (from 1970 in the comprehensive school and in 1973 in the senior secondary school). There are seminar reports and articles on the curriculum reform but they mostly concentrate on describing the changes and contents of the new curricula. Real research into the factors influencing the processes of reform has not been done. The foundation provided by Nordic co-operation has been widely recognized as an important point in choosing subject matter but it is also true that the new plans had been quite widely implemented in the elementary level of the comprehensive school (grades 1-6) as well as in the senior secondary school before the reform. The contribution of those experiments cannot be underestimated.

3. The influence of curriculum reform on learning outcomes

In Sweden Margareta Kristiansson has, in her doctoral thesis (1979), compared the learning outcomes attained during the curriculum of the year 1962 to those attained after the implementation of the new curriculum 1969. In the latter the subject matter of mathematics had been reformed according to the quiding principles of Nordic co-operation. Experiences of its application were scarce at the time the investigation was made at the beginning of the 1970s. Teachers had been slow in changing their teaching and the results revealed no essential changes except in grade 6 where the temporary implementation of the new curriculum induced worse learning outcomes than before. Attitudes towards mathematics teaching did not change during the investigation. Differences between urban and rural schools were greater than those between different curricula.

The empirical part of Kristiansson's reserach is carefully realized but potential explanations for the changes in learning outcomes remain superficial since no information of teaching situations is available. In the reseracher's opinion the decisive factor is not the content of the curriculum as such but the way

teaching has been arranged. Many other investigations reveal that the written curriculum is of little significance to learning outcomes.

In Finland, too, comparisons were made between the outcomes of the elementary and comprehensive school when the curriculum for the comprehensive school was being implemented. According to Solanti's report (Solanti 1972) differences remained small when some pupils were using textbooks meant for the elementary school ant others those for the comprehensive school. In some theses teachers made it clear that the new subject matter induces learning difficulties but on the other hand, it has been possible to facilitate the introduction of set theory with remedial teaching.

Long-term comparisons have been widely practised within the project of International Evaluation of Achievement. The first part of the investigation was realized in 1964 and the second in 1981. The investigation was aimed at 13-year-old pupils independently of school type and at pupils about to finish the senior secondary school. A great many undifferentiated tasks were presented on both occasions. Acoording to a preliminary report (Kangasniemi 1983), very few changes have taken place during this time in terms of the achievement level of undifferentiated tasks. Changes tend to be contradictory so as to express the changes in the curriculum content which have taken place during that time.

A wide national survey of the teaching of mathematics in the comprehensive school was made in Finland in 1979. The investigation concentrated especially on essential cognitive goals and achievement was compared, for wxample, to the outcomes of the firs stage of the IEA-project from 1964 (Kupari 1982). The results revealed that pupils' cognitive level had become significantly more extensive although it had risen only in terms of some sub-areas. For example, after the elementary level of the comprehensive school (grades 1-6) pupils' ability to

calculate was of about the same level as that of pupils' during the old parallel school system in 1964. On the other hand, pupils from the comprehensive school did not do as well in tasks requiring applied knowledge. The results of this study, too, reflected the changes in terms of the goals and contents in the curriculum. Furthermore, it was found out that pupils' attitudes had developed in a positive direction compared to the prior school system.

The survey of 1979 on mathematics teaching in the comprehensive school also paid attention to the outcomes in relation to teachers' level of aspiration - in other words, in relation to those goals that teachers considered should be attained by the whole age-group (Kupari 1983). The outcomes obtained were nearly as good as teachers regarded as attainable if the syllabi and instruction remain about the same as before. Only the outcomes of the lower set of the secondary level of the comprehensive school (grades 7-9) remained below teachers' level of aspiration. On the other hand, however, the pupil who had attained the objectives with the lowest possible marks did not master more than half of the learning contents required in the elementary level and a third in the secondary level of the comprehensive school. This finding does not appear satisfactory in terms of the mastery of basic subject matter. The above results are not detailed and hence to be interpreted with reserve. The investigation has, however, revealed important questions concerning the evaluation of the curriculum and mathematics teaching.

Many small-scale studies have been done in the 1970s of the attainment of the goals presented in the mathematics curriculum. Investigations dealing with learning difficulties include a lot of information about the sub-areas of the curriculum which have proved out to be difficult for the pupils. It is not easy, however, to draw any conclusions on that basis of the effects of the curriculum reform. One of the conclusions to be drawn is that new subject matter is fairly well acquired after teachers have adopted an appropriate way of presenting it. Set theory and

logic were regarded as two separate sub-areas at the planning stage of the reform at the end of the 1960s and the outcomes attained varied to a great extent. First, pupils learnt to write down numerous new symbols but they did not know how to apply the conceptual schemes of set theory and logic. Later, these matters were included in the mathematical code in a more simple form and hence teaching became easier although less systematic. At present the new subject matter brought along by the "modernization" of mathematics is fairly well acquired although it seems that the development of higher processes of thinking through set-theory and logic still remains inconsiderable. It is difficult to form a general view of the effects of the curriculum reform through reserach.

4. Summary of research results

Only the investigations in the Nordic countries that directly deal with curriculum reform of mathematics and its effects have been taken into consideration in the above survey. Research on mathematics teaching has greatly increased in all of the countries in the 1970s, perhaps most in Finland where there are assistant professorship in mathematics, physics and chemistry, too. Sweden, on the other hand, has most groups of scientists. However, curriculum reform has not attained scientists' great interest in the Nordic countries, neither at the planning stage nor after its implementation. Many countries in Central Europe have permanent groups of scientists for this purpose. In addition to curriculum design they take part in reforming teacher training and developing learning materials.

According to research, the reform of the curricula for mathematics has taken place as a result of fairly incoherent processes. If the process of decision-making is highly centralized, like in Sweden, reform remains superficial. Issues of educational policy have contributed to the fact that changes are imposed rapidly without adequate, systematic developmental measures. Hence,

teachers' readiness to implement reforms remains low.

After the "modernization" of mathematics teaching there was criticism against the reform. Especially the decline in numeracy was disapproved of. This kind of trend could be seen in the outcomes but no permanent decline since mathematical knowledge has become more versatile at the same time. Calculations with decimals and fractions have been left in a secondary position as non-equations, vectors etc. are introduced even in the comprehensive school. It is possible to note afterwards that the textbooks for teaching algebra by Väisälä in the 1950s in Finland were in many respects modern. Keeping this in mind we could say that the reform was not as abrupt in Finland as in many other countries.

Was the reform unsuccessful as Håstad claimed in his doctoral dissertation? One may come to a conclusion like that with such a predisposition. Professor Tamas Varga, the experienced Hungarian scientists whose main interest is mathematics teaching, pointed out that it will take 30 years to implement a reform like this. After a confusing beginning the situation has become established in terms of the subject matter. Problems are mostly caused by changes of educational policy like the abolition of setting in the secondary level of the comprehensive school (grades 7-9). There is no reason to return to the old system - "bact to basics" - but to search for a more elaborate way of instruction on the basis of the present reform - "forward to fundamentals" in the words of Prof. Tamas Varga.

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COGNITIVE PROCESSES AND STRATEGIES IN ELEMENTARY MATHEMATICS

Tapio Keranto

1. Introduction

The most recent research in the West into the learning and teaching of mathematical subject matter has been stimulated principally by cognitive psychology and tne paradigms and research findings of Piagetian research tradition (see e.g. Beth & Piaget 1966, Case 1980, DeCorte 1980, DeCorte & Verschaffel 1981, DeCorte et al 1982, Keranto 1981; 1982; 1983a, Klahr 1976, Klahr & Wallace 1976, Leino 1978; 1981; 1982, Newell & Simon 1972, Neisser 1967). From the information processing standpoint the aim is to analyse the inner processes and other factors which underlie and affect individual's external behavior. To put it more precisely, the goal is to find out what part functions and factors are connected with mathematical problem-solving and what role they paly. It is also noticeable that the content matter of research is becoming increasingly concerned with what goes on in the school and teaching matter. At the same time, the weaknesses of Piaget's structuralism have become apparent as well as its remoteness from the usual mathematical problems encountered in the work of the school and their associated cognitive and motivational processes and factors.

The Piagetian tradition empahasizes the significance of prenumerical mathematico-logical operations in the acquisition of mathematical concepts and principles. The relationship of

prenumerical operations with information processing capacity and with number-listing and the linquistic and social foundations of use has remained largely unexplored. In the light of recent studies it appears more and more clearly that skills connected with number-listing and the search for the significance of number words play a significant role in learning to understand mathematical concepts and principles (see e.g. Fuson1981, Fuson & Hall 1982, Fuson et al 1982, Gelman 1972, Gelman & Gallistel 1978, Keranto 1981; 1983a; 1983b). This points in a similar to that of the Soviet research paradigm and findings (Davidov & Andronov 1979, Galperin 1957, Galperin & Georgiev 1969, Vygotsky 1962). Indeed, the movement emphasizing information processing in the learning and teaching mathematics has brought Soviet and Western research thinking closer to each other. There is anincreasing interest in the cognitive and affective processes of the normal classroom situation and their interrelationship (see e.g. Aebli 1963; 1967; 1980, Aebli et al 1968, Keranto 1983b, Nicholls & Miller 1983, Salonen et al 1982, Weiner 1979).

The present investigation is an experimental research on the interrelationships between cognitive and affective factors which underlie and influence the arithmetical and measurement performance of first-grade pupils. The investigation is based on a longitudinal study of the development of mathematical thought processes and strategies begun in autumn 1982. The first stage of study attempted to chart the interrelationships of the following cognitive factors:

A. The operationalisation of the developmental stages used by Piaget or Piaget-abilities (understanding of transitivity, the "intensive" relationship of a set and its subset, conservation of quantity (see Beth & Piaget 1966, Brainerd 1978; 1979, Flavell 1963, Keranto 1981; 1983a, Piaget 1952)
B. Information processing capacity in numerical contents (see Case 1972; 1974; 1977; 1980, Keranto 1983a; 1983b, Pascual-Leone 1970)
C. Number-listing skills (listing upwards and downwards from

different points and by different amounts (see Fuson 1981, Fuson & Hall 1982, Fuson et al 1982, Keranto 1981; 1983a; 1983b)

D. Addition and subtraction skills and their associated counting strategies (see Carpenter et al 1981; 1982, Fuson 1981, Fuson & Hall 1982, Keranto 1983a;1983b)

E. Measurement skills and their associated strategies (see Bailey 1974, Carpenter 1971; 1975, Galperin & Georgiev 1969; Hiebert 1979; 1981, Inhelder 1972, Keranto 1983a, Piaget et al 1960, Steffe 1971)

The second stage of the longitudinal study sought to ascertain the development of skills associated with points B.-D. over the first school year and charted their relationship with the following factors in mathematics learning:

F. Study achievement in mathematics in the first school year

G. Motivation for and attitudes towards mathematics study

The experimental investigations of motivation and attitudes in mathematics study were for their part linked to previous research on attitudes (Malinen 1969, Leino 1977, Karjalainen 1982), cognitive psychology (Madsen 1966) and work and performance motivation (Atkinson 1964, Chung 1977, McClelland et al 1953, Ruohotie 1982a, b)

2. Experiment I

The main question to which the experimental section I addresses itself are as follows:

1. How do Piagetian abilities relate to the individual's performance on and strategies involved in the central (from a practical point of view) tasks of addition and subtraction?
2. What is the relationship of information processing capacity to Piagetian abilities and to arithmetical and measurement performance and strategies?

3. How do sequence skills relate to performance on and strategies used in addition and subtraction tasks to Piagetian abilities, and to information processing capacity?
4. Are there hierarchies of sequence skills, and how can these be put in to use in basic mathematics teaching?
5. What are the dependency relationships between the various subsectors of the cognitive field under investigation on a macrolevel?

Method

The subjects were interviewed individually. Each child's performance and strategies used were recorded both in a written code and on tape. The measurements were carried out in the training school at the University of Tampere Teacher Training College in Hämeenlinna (25.8.82 - 15.10.82). All first-graders (N=35) took part in the measurements, their ages ranging between 6.8 and 7.8. Each child was interviewed four times, each interview lasting 35 - 45 minutes depending on the child's level of ability and type of test. To sustain motivation and avoid tiredness the tests included breaks for relaxation. The tests were conducted throughout by the researcher. The order in which each child carried out the tests was: sequence skills - arithmetical addition and subtraction abilities - information processing capacity - prenumerical operations and measurement abilities.

Measurement

The types of tasks used are outlined below, with the abbreviations used and the number of items given in parentheses.

Piagetian abilities: transitive judgment (TR/4); (SUBS/3); functional seriation (SER/2); inclusion (IN/3-6); conservation of ascertained one-to-one correspondence (EJEC/4); number consevation under identity conditions (NC/2); and conservation of length under conditions of indentity and equivalence (LC/3) (Cf. Brainerd 1978, 1979; Piaget 1952; Piaget et al 1960).

Information processing capacity: Enumeration Span when all numbers the subject must count up to are given in advance (ESP/10); Enumeration Span when numbers the subject must count up to are given one by one (ESP(0)/5 ; a modification of Case's CSP measure); Counting Span (CSP/3; Case's CSP measure) (Case 1972, 1974, 1980; Case et al 1979).

Sequence skills: A. Forward sequence: fluent and accurate enumeration (ENUM/1); "count" up from x (UP/5); "count" up to x (TO/2); "count" up from x to y (UPTO/3); "count" up from x by "N" (UPBY/6); "count" up from x by the same number m (SABY/5). B. Backward sequence: "count" down from x (DO/2); "count" down from y to x (DOTO/3); "count" down from y by "N" (DOBY/6). C. Relational skills: "number" after x (AF/3); "number" before x (BEF/3); "numbers" between x and y (BET/3); "N" bigger/smaller than x (BS/8). (Fuson & Hall 1982, Keranto 1981).

Arithmetical and cardinal abilities: Combine plus (COBP/3); Join plus (JOIP/3); Separate minus (SEPM/3); Join minus (JOIM/3); Reversible join minus (RJOIM/3); Combine minus (COMB/5); Compare (COMP/4); accurate counting (COUNT/2); cardinal principle (CARD/2); the formation of equivalence and inequivalence (EJE/3); progressive summation (PRS/3) (Carpenter et al 1982; Gelman and Gallistel 1978; Keranto 1979, 1981).

Measurement abilities: the "covering" of length and the inverse relation of measurement number and unit of measurement (COV/4); Piaget's subdividing a straight line, measured by "motion" along a "car track" (MOT/3); "pedalling" tasks or travel (TRAV/3); measurement technique tasks (MT/2) (Bailey 1974; Carpenter 1971, 1975; Hiebert 1981; Piaget et al 1960 pp. 128 - 149; Steffe 1971).

Scoring

Two systems were used, one based on correctness of performance and the other on the subject's conciousness of the process and strategies employed. These yielded characteristic sum

variables for each task type. The variables based on performance and the "qualitative" variables based on process and strategies proved to behave similarly in correlation and regression analyses carried out. However, it was noticeable that performance variables in respect of Piagetian abilities did not have the same degree of explanatory power as the corresponding process variables. In other words, the criterion that the subjects had to be able to explain the way they arrived at their answers had a significant effect on the explanatory power of Piagetioan variables in relation to other variables in the cognitive field. This was understandable in view of the fact that there was little variance on the TR, SUBS, IN and SER variables on performance scoring, since success on these tasks was almost 100 %. By demanding that subjects should explain their answers, however, it was possible to score 0, 1 or 2, where 2 represents a level on which the calculation process is correct and the explanation adequate. The results of the study, to which we shall now turn, lend support to the use of Piaget's set of criteria in the definition of operational thinking.

Results and Discussion

As regards question 1, Piagetian abilities as assessed on process criteria in particular, appear to have predictive power with respect to arithmetical performance and especially strategies and thus to be closely connected with these. Piagetian abilities assessed on performance had a predictive power 10 - 30 % lower; hereafter it will be to process measurement that reference is mad when Piagetian abilities are mentioned. The predictive power of Piagetian ablities in respect of arithmetical strategies varied between 22 % (RJOIM/TR) and 61 % (COMP/SUBS and EJEC); and in respect of arithmetical performance between 12 % (JOIM/TR) and 56 % (COMP/EJEC and LC). One general observation was that performance on tasks closely associated with addition, COBP and JOIP were poorly explained whereas the more difficult types COBM and COMP were better explained by Piagetian abilities. This suggests that perfor-

mance on the simplest tasks is not so dependent on the level of development of Piagetian abilities, whereas performance on tasks involving quantative comparison of sets is. This can be explained in terms of the development of strategies, in which Piagetian abilities would appear to play a significant role.

In an attempt to throw light on question 2, four information processing capacity (IPC) measures were tried out as described earlier. These measures were designed so as to take account of earlier studies and the most central issues in this study, with Case's reserach (1972, 1980) as a point of departure. The most reliable of these measures proved to be the ESP measure, with a reliablity coefficient of 0.79. This measure was similar to the BDS measure used by Case but was regarded as more in correspondence with the pressures of real-time processing the child has to cope with in keeping various things in mind simultaneously on sequence and arithmetic tasks (cf. Case 1972). The span values of beginning schoolchildren were found to be typically 2 or 3, which correspond to those reported by Case (1972) and Pascual-Leone (1970). On the other hand, it was observed that span values depended a great deal on how large the numbers were in each task and particularly on the specific content. The degree of operational automation was observable when span values on tasks involving sequence contents were compared with those on tasks relating to counting and measurement contents; the latter were considerably lower. These differences were further noticeable in the examination of how far Piagetian abilities, sequence skills and arithmetic and measurement abilities can be explained by IPC abilities.

The following nine variables were formed relating to IPC abilities: ESP, ESP(S), ESP(O), CSP, MEASP, $ESP_m(S)$, $ESP_m(O)$, CSP_m and $MEASP_m$. Variables ESP(S) to MEASP refer to mode performance on each test, and those with sub-index $_m$ to maximum performance. The ESP(S) variable is obtained from the ESP test, whilst ESP is the sum variable of the items in this test;

$0 \leqq$ Range (ESP) $\leqq 10$. As regards regression models, only those were chosen in which all variables increased the predictive power by at least 5 %. Here it may be noted that all t-tests used in the study were consistent with the regression models and do not add much to the present discussion; they will not therefore be treated any further in this summary.

The predictive power of IPC abilities in relation to arithmetic strategies varied between 13 % (SEPM/ESP) and 46 % (COMP/ESP and CSP_m), and in relation to arithmetic performance slightly lower at between 14 % (SEPM/ESP) and 40 % (COMP/ESP). A different picture emerged, however, as regards the relation of IPC abilities to measurement abilities. Here the best predictors were the MEASP and CSP variables. Levels of predictive power ranged from 12 % (MT/ESP_m(S)) to 40 % (MOT/CSP, ESP(O) and MEASP).

A general conclusion to be drawn would seem to be that IPC abilities are closely tied to specific contents, and that there is therefore a need to develop content-specific IPC measures further before the real significance of span values to the learning of particular contents can be seen.

Questions 3 and 4 are closely enough related to be treated here together as a whole. Some of the problem areas involved have been dealt with in the above presentation, too. Sequence skills together with arithmetic processes and strategies in addition and subtraction tasks formed the central core of this study. The choice of sequence skills was based on previous research by Fuson and Hall (1982) and by the present writer (Keranto 1981).

The development of hierarchies of sequence skills was investigated in parts using Guttman's scale (Guttman 1944; White & Saltz 1957). "Complete" hierarchies were obtained in particular in the case of forward and relational skills. The best hierarchy in the 1 - 10 range proved to be UP - TO - UPTO - UPBY (CR =

0.96, MMR = 0.82 and PPR = 0.78). In the 10 - 20 range the best and literally complete hierarchy was AF - BEF - BET (CR = 1.00, MMR = 0.76 and PPR = 1.00). The formation of hierarchies was complicated by the developmental relations of UPTO and UPBY skills. Similarly, DOTO and DOBY skills "spoilt" the hierarchy of backward sequence skills in the 1 - 10 range. However, in the 10 - 20 range the following hierarchy was obtained: DOBY"2" - DO - DOBY"5" - DOTO - DOBY"8" (CR = 0.92, MMR = 0.66 and PPR = 0.76) The decisive factor here, in other words, was the number of digits by which the subjects had to count back from a given number. The foregoing hierarchies suggest a possible systematic development of sequence skills. This is an important point, since sequence skills prove to be sound predictors of arithmetic performance and strategies, as both the logical analysis carried out and the experimental results indicated.

The predictive power of sequence skills in relation to arithmetic performance ranged between 22 % (JOIP/SABY) and 86 % (RJOIM/BEF, UP, TO, DO); the COBM and JOIM variables, too, were high predictors at 72 % (COBM/DOBY) and 73 % (JOIM/BEF, AF, ENUM). In relation to arithmetic strategies, predictive values were even higher, ranging from 45 % (COBP/SABY and BET) to 84 % (COBM/DOBY, ENUM, SABY, UPTO). The best predictor as regards strategies on problems of the COBP, JOIP and SEPM types was the SABY variable, i.e. the ability to coup up by a given number, the figures being 45 %, 57 % (as also with UPBY) and 49 % respectively. The best predictor of JOIM and RJOIM strategies was the BEF variable, i.e. the ability to state a number's immediate predecessor, with values at 68 % (JOIM/BEF, UP, UPTO) and 78 % (RJOIM/BEF, SABY, AF). Finally, the best predictor of COMP abilities was the BS variable (59 %), which is logical enough in that COMP and BS problems correspond in their deep structure.

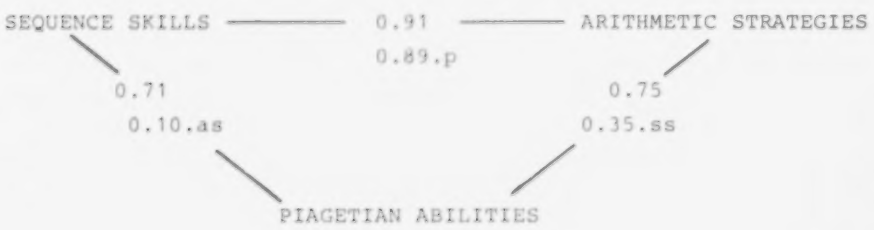
Piagetian ablilities as predictors of sequence skills generally had values below 50 %, but in the case of the important BEF, UPBY and DOBY skills there were figures of 58 % (BEF/SER, LC,

IN and NC), 61 % (UPBY/LC and NC) and 62 % (DOBY/LC, NC and SUBS). It was thus conservation variables which proved to be the best predictors UPBY and DOBY skills, and this indicates that these more demanding sequence skills are bound up with the individual's level of operational development.

When Piagetian abilities and IPC abilities taken together were assessed as predictors of sequence skills, values of over 50 % were generally obtained, the highest being 70 % (DOBY/LC, NC, ESP and SUBS). It is thus evident that cognitive abilities (Piaget and IPC) play a significant role in the acquisition of sequence skills, which are in turn closely connected with arithmetic strategies. The route by which Piagetian abilities influence sequence skills emerges at the macro-level in the following discussion, which also throws light on other dependency relations in the cognitive field.

To answer question 5, sum variables were formed from the performance and process variables for each sub-sector of the cognitive field under investigation. The following "triangle" reveals the macro-level dependency relations between Piagetian abilities, sequence skills, and arithmetic abilities:

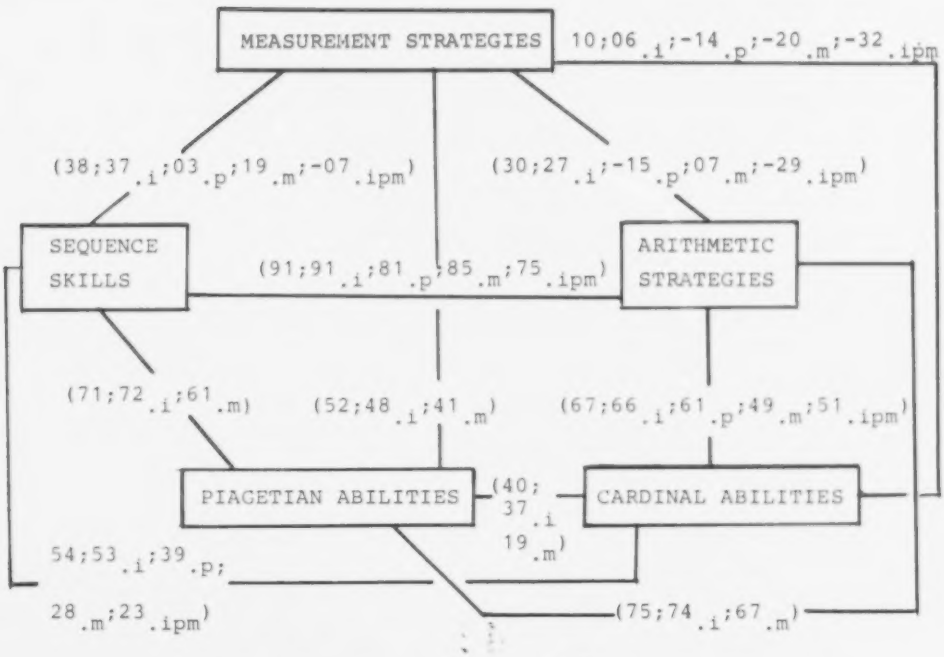
Figure 1. A model of the dependency relationships between sequency skills, arithmetic strategies and Piagetian abilities (N=35); .as, .ss and .p indicate partial correlations where arithmetic strategies, sequence skills and Piagetian abilities constant respectively



The relationship between Piagetian abilities and sequence skills is explained by the arithmetic strategies used. This is in keeping with the earlier detailed analyses: Piagetian abilities were found to be very good predictors precisely of arithmetic strategies. Again, it was stated that both on logical grounds and experimentally arithmetic strategies are closely related with the more developed sequence skills, so that it is not surprising that on the macro-level, too, the variance shared by sequence skills and Piagetian abilities falls from 50 % to 1 % when arithmetic strategies are constant.

The following figure presents a larger model of the cognitive field investigated:

Figure 2. The dependency relationships between sub-sectors of the cognitive field studied (N=35); .i, .p, .m and .ipm indicate partial correlations where order of age, Piagetian abilities and memory (IPC) are constants separately and all together respectively



The highest correlation was that between sequence skills and arithmetic strategies where cognitive abilities were constant, which reflects the close connection found on the specific level. The correlation of Piagetian abilities with sequence skills and arithmetic strategies remains high even when memory is taken as constant. Their correlation with measurement strategies, too, remains at a statistically significant level with memory constant. Sequence skills appear to be connected with measurement strategies and cardinal abilities through cognitive abilities, while cardinal abilities connect with Piagetian abilities via memory. It can be observed that the measurement abilities studied are relatively independent of and thus operationally different from the other parts of the cognitive field with the exception of Piagetian abilities. The results charted in the above diagram were also reflected in regression analyses performed between macro-variables. The most significant predictors of arithmetic strategies were, in accordance with the model, sequence skills, cardinal abilities and Piagetian abilities in that order. The level of predictive power was as high as 89 %. On the basis of this it is reasonable to speculate that the factors determining the development of arithmetic strategies are largely localised. As expected, the only significant predictor of sequence skills was arithmetic strategies, the predictive power high at 83 %. The best predictors of Piagetian abilities were arithmetic strategies and measurement strategies (66 %); of cardinal abilities, arithmetic strategies (45 %); and of measurement strategies, Piagetian abilities (27 %). All these results are consistent with those recorded on the specific level.

3. Experiment II

The main questions the experimental part II of the study set out to examine were as follows:

1. To what extent and in what way do sequence skills, arithmetical strategies and information processing capacity develop during the first school year?
2. To what extent and in what way do sequence skills, arithmetical strategies and information processing capacity relate to study achievement in mathematics?
3. To what extent and in what way do attitudes towards mathematics learning relate to first-graders' arithmetical processes and achievement in mathematics?

So the experimental investigation of attitudes to mathematics study sought further to throw light on the role played by cognitive and affective factors in mathematics achievement.

Method

The factors associated with arithmetical processes (sequence skills, arithmetical strategies and information processing capacity) were measured in individual interviews in May 1983. All first-graders in one school (N=35) took part in the measurement, their ages ranging between 7.6 and 8.6. The standardised interview for one pupil lasted 35-45 min. depending on the pupil´s arithmetical abilities, sequence skills and information processing capacity. The tests were conducted throughout by the author. The processes and strategies used by each child were recorded in a written code. Attitudes were measured by means of a straightforward multiple choice technique involving three alternatives (disagree-can´t say-agree). The same measure was administered to the pupils twice, with an interval of three days intervening, in May 1983. In this way it was hoped to establish the reliability and validity of each attitude-measuring item and the consistency of pupils´ responses. All first and second-graders (N=35+32=67) took part in the measurement. Each measurement session took approximately one lesson (=45 min). The responses of the second- graders were used in assessing reliability and establishing attitudinal dimensions. This made the number of observations large enough for factor analyses to be carried out.

Measurement

The types of tasks used in the measurement are outlined below, with the abbreviations used and the number of items involved given in parentheses. The majority of items relating to arithmetical processes were identical with those used in the measurements carried out in autumn 1982 (Keranto 1983a).

Sequence skills: "count" up from x by "N" (UPBY/6); "count" down from y by "N" (DOBY/6); "count" up from x by the same number m (SABY/5) (cf. Fuson & Hall 1982, Keranto 1983a).

Arithmetical strategies of the types: Join minus (JOIM/2); Reversible (RJOIM/2); Combine minus (COBM/6) (cf. Carpenter et al 1982, Keranto 1983a).

Information processing capacity: Enumeration Span when all numbers the subject must count up to (from 1) are given in advance (ESP(S)/13) (cf. Backward Digit Span (BDS), Case 1972, 1974, Keranto 1983a).

Mathematics achievement: mental arithmetic in the range 0 - 20, written addition skill, written subtraction skill (MENTAL1, PLUS1, MIN1); mental arithmetic, written addition skill, written subtraction skill in the range 20 - 100 (MENTAL2, PLUS2, MIN2); verbal task solving skill (SOLV). The above were measured on a five-point scale (weak, passable, satisfactory, good, very good) and determined on information supplied specifically for this study by the firstgraders' teachers.

Attitudes towards mathematics study: difficulty experienced in mathematics learning (DIFF/3); necessity (NECES/3), willingness to try (TRY/6); experience of success (ESUC/3); cognitive and functional needs (KOGN/4 and FUNC/4); social needs (SOC/4); fear (FEAR/3); aggressiveness (AGGR/2); desire to have more/fewer hours (DESIR/2); (un)attractiveness of mathematics as compared with other subjects in the basic curriculum (ATTR).

Scoring

For the sequence skills UPBY and DOBY the scoring was based on the performance process as follows: 0 where no adequate strategy; 1 where the strategy involves external aids or number substitution; 2 where the strategy is either "calculate in advance which number is to be counted up to" or "keep a count of each number spoken". As for the SABY task, the scoring system was: 0 for failure to list the two following numbers; 1 where the listing of the intervening numbers helps; 2 for a direct, fluent response without listing the intervening numbers.

In the case of the arithmetical tasks JOIM, RJOIM and COBM the scoring was again based on strategies as follows: 0 where no adequate strategy; 1 where there is a resort to external aids, fingers, blocks or other visual representation; 2 for a protracted mental calculation; 3 for performance based on factual knowledge.

Sum variables relating to the above tasks were formed as follows:

$$UPBY=\sum_{i=1}^{6}UPBYi,\quad DOBY=\sum_{i=1}^{6}DOBYi,\quad SABY=\sum_{i=1}^{5}SABYi,\quad JOIM=\sum_{i=1}^{2}\underline{JOIMi}+\sum_{i=1}^{2}RJOIMi,$$

$$COBM=\sum_{i=1}^{6}COBMi,\quad SEQU=UPBY+DOBY+SABY,\quad ARITME=\underline{JOIM}+COBM$$

This formulation was supported by correlation and reliability studies.

In the case of information processing capacity two variables were formed. The ESP variable recorded the number of successfully completed tasks on the ESP(S) test ($N\leq 13$). The SPAN variable values were determined as follows: 0 or 1 where the pupil fails to reach the next level (no cases); 2 for more than half right on tasks involving the retention of two units of information (5 items); 3 for fulfilment of the previous criterion and more than half right on tasks involving three

units of information (5 items); 4 for fulfilment of the previous criterion and over half right on tasks involving four units of information (3 items). In the case of every subject the criterion in question was fully satisfactory.

The mathematics achievement five-point variables MENTAL1-2, PLUS1-2, MIN1-2 and SOLV were formed into the following sum variables: MENTAL=MENTAL1+MENTAL2; PLUS=PLUS1+PLUS2; MIN=MIN1+MIN2; CSKILL1=MENTAL1+PLUS1+MIN1; CSKILL2=MENTAL2+PLUS2+MIN2; CSKILL=CSKILL1+CSKILL2+SOLV. School grades at the end of the spring term yielded information on the pupils' general performance in mathematics (MAT), reading (READ), writing (WRITE) and manual skills (MANUAL). A three-point scale was used: 0 needing help; 1 fair; 2 good. Grades were also given for school behaviour, specifically good behaviour (BEH), conscientiousness (CONSC) and attention to detail (ATTEN) with the three-point scale here: 0 unsatisfactory; 1 satisfactory overall; 2 regularly good. From these three-point variables were formed the following sum variables: LANGU=READ+WRITE; SUCCESS=MAT+LANGU+MANUAL and BEHAV=BEH+CONSL+ATTEN.

Attitude sum variables were also formed on the basis of correlation and factor analyses and reliability studies of the individual attitude variables: EMOTIO=AGGR1+AGGR2; TRY=TRY1+TRY2; DIFF=DIFF1+DIFF2+DIFF3; ESUC=ESUC1+ESUC2+ESUC3; FEAR=FEAR1+FEAR2 and KOGN=FUNC1+KOGN1+FUNC3+KOGN3. The responses on each attitude test item were scored on the three-point scale: 0 disagree; 1 can't say; 2 agree.

Results and Discussion

Findings with regard to question 1: Binomial tests (see Siegel 1958) revealed an extreamly significant ($P<.001$) development of UPBY and DOBY skills during the first school year. The vast majority of those children (73 - 100 % depending on the task) who had been unable to perform such tasks on starting school were able to do them by spring using finger-counting or mental strategies. Again, those who on starting school had used

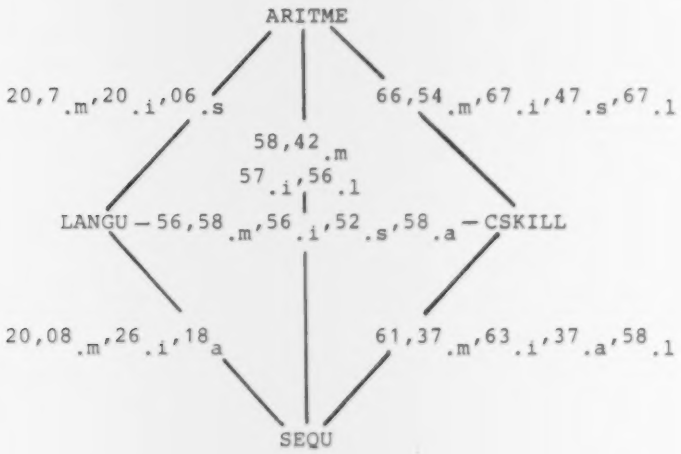
finger-counting had most of them (67 - 100 % depending on task) gone over to the mental strategy "calculate in advance which number is to be counted up to". The consistency of memory strategies over the whole UPBY-DOBY field was high at 83 %. Reliability figures for the sequence skills UPBY, DOBY and SABY were also high (.77-.99). As in earlier studies, the development of arithmetical strategies was investigated in connection with the Join minus, RJoin minus and Combine minus task types. Binomial tests revealed an extreamly significant transfer from the use of external aids to the use of more developed mental strategies. In fact, of the pupils using external aids or protracted mental strategies in the autumn term the majority (72 - 87 % depending on the task) had switched over to applying factual knowledge by spring. In the case of the Combine minus-type tasks, this meant that the pupils in question had applied the number distributions they had learned in school in determining an unknown number. Of the pupils who were unable to do Combine minus tasks in the autumn, most (50 - 65 % depending on the task) employed finger-counting strategies and some had achieved the level of factual knowledge. The extent to which resort was made to external aids increased as the size of the unknown and/or the known numbers increased. These findings strongly support the view that external aids are significant and indispensable in the learning and teaching of basic mathematics skills (cf. Keranto 1979, 1981, 1983a). The consistency of the Combine minus task type was found to be high (80 - 94 % depending on the number range), whereas the consistency for Join minus and RJoin minus was rather lower (71 and 49 %). Reliability figures were of much the same order: .74-.94 for Combine minus, but .51 and .65 for Join minus and RJoin minus respectively. The lower values most likely resulted from the differences in the number ranges involved in the Join minus and RJoin minus tasks.

As noted earlier, information processing capacity was measured by means of an ESP(S) measure involving number sequence contents on the basis of earlier research (Enumeration Span in Simultaneous Systems, Keranto 1983a, 47-52). This was a deve-

lopment of Case´s BDS measure (Backward Digit Span, 1972). Reliability figures ranged between .61 and .73. Binomial tests showed an extremely significant ($p<.001$) development of information processing capacity during the first school year. In fact, all the first-graders had achieved at least the level of two-unit retention (span=2) by spring. In other words, these children were able to hold in mind at least two numbers words given in advance as the counted up from 1 first to one number then to the other. The numbers were chosen from within the range of numbers the subjects could fluently list.

Findings with regard to question 2: The assessment of the pupils´ achievement made by the teachers specifically for this study proved consistent and of high reliability (.90-.95). Stepwise regression analyses ($p<.05$ for both t and F values) indicated that sequence skills and arithmetical strategies accounted for 52 - 59 % of the variance on the various components of mathematics achievement. The best predictors were found to be mental arithmetic skill (PAA) and verbal task solving skill. Including the information processing capacity variables ESP and SPAN among the predictors increased the predictive power only in the case of verbal task skill (SAN) to a figure of 66 %. The figures obtained are in any case high and exceed in some degree the average 50 % figures for predictive power anticipated in Bloom´s theory of school learning (cf. Bloom 1976, Lahdes 1980, 1982). Predictable, sequence, arithmetical strategies and information processing capacity were not found to be connected to any statistically significant degree with mother tongue achievement. This did not mean, however, that there was no relationship between reading and writing ability and mathematics achievement and counting skills. Indeed, a relatively specific and statistically highly significant relationship was found between mother tongue achievement and the various components of mathematics achievement. This can be seen in more detail in the figure below, which shows the interrelationship of the sum variable CSKILL with the SEQU, ARITME and LANGU variables.

Figure 3. A model of the interdependency relations of counting skills (CSKILL), sequence skills (SEQU), arithmetical strategies (ARITME) and mother tongue achievement (LANGU), where .m, .i, .s, .a, .l stand for IPC, age, SEQU, ARITME, LANGU constant (N=35)



In the regression models mother tongue achievement together with sequence skills, arithmetical strategies and information processing capacity predicted 53 - 67 % of the variance on the various components of mathematics achievement. The best predictors were once again mental arithmetic skill and verbal task solving skill. This is logical, since sequence skills, arithmetical strategies and information processing capacity were measured under conditions of oral and mental operation.

Findings with regard to question 3: In determining the dimensions of the attitude measuring instrument factor analyses had been used, which had yielded a theoretical division and a six-factor structure, which corresponded well to earlier measurements of attitudes in mathematics learning (overall predictive power 46.5 %) (cf. Leino 1977). Similar findings emerged from reliability studies based on both factor analyses and re-test correlations (re-test correlations generally

$r \geq .53^{***}$). The variables EMOTIA, DIFF, FEAR, DESIR1-2 and ATTR formed a relatatively reliable whole. With the TRY and NECES items there was the problem of little deviation (nearly all the pupils "wanted to try" and thought that mathematics study was necessary). Comparing the first-graders' answers with those of the second graders tested on the same battery, it was found out that the first-graders responses on the KOGN an ESUC variables were more opaque than the second-graders'. An interview would have been necessary in addition to the questioning procedure in order to have ascertained how the children interpreted the questions related to the above-mentioned variables. The same applies to the items measuring social needs (SOC1-4), which represented the most disparate set, and the most problematic as regards content validity, in the whole battery. In relation to reliability, the consistency of individual pupils' responses over two measuring sessions was investigated. It was found out that consistency bore a statistically at least nearly significant relation ($p<.05$) to sequence skills, arithmetical strategies, mathematics achievement and difficulty experienced (DIFF). In other words, the weaker the pupils the more changes in response occurred between the two measuring sessions.

T-tets showed that difficulty experienced (DIFF) and attractiviness (ATTR) were best and most widely correlated with sequence skills, arithmetical strategies and mathematics achievement. Variables FEAR and ESUC were also correlated with cognitive variables. The results indicated in the t-tests were also naturally reflected in regression analyses carried out. Attitude variables predicted 14 - 45 % of the variance in sequence skills and arithmetical strategies. The best predictor in the regression models was difficulty experienced (DIFF). A similar finding was made by Leino among upper comperhensive school pupils (Leino 1977). As regards the various components of mathematics achievement, attitude variables predicted 31 - 40 % of the variance. Sequence skills, arithmetical strategies and attitude variables taken together predicted 52 - 67 % of

the variance on the components of mathematics achievement. When in addition the information processing capacity variables ESP and SPAN were included among the predictors, the predictive power rose as high as 79 % in the case of mental arithmetic and verbal task solving skills; and figures were generally high (69 - 79 %), exceeding the 65 % average level anticipated by Bloom (Bloom 1976).

Although the results obtained appear for the most part reliable and in accordance with previous experimental findings and theoretical studies, they must still be treated with caution. This is especially the case with the attitude measurements, which in this study were tentative in nature, seeking to discover connections with cognitive processes. Future research could make use of interviews, observation and long-term procedures in the assessment of motivation and attitudes. Only then will it be possible to obtain a more detailed picture of how motivation in mathematics learning relates to attitudes, the self-view, and cognitive processes.

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THE IMPACT OF THE COGNITIVE PROFILE ON DIFFERENCES IN LEVELS OF ACADEMIC ACHIEVEMENT

Kaarina Aimo
Marja-Leena Viilo

1. Introduction

It is the qualitative goal of education to help the learner to achieve general educational goals and to develop him in as many areas as possible according to his individual characteristics and talents. Attaining these goals has involved a great many problems, especially with slow learners and students with learning difficulties.

Different learning strategies have been used in order to improve academic achievement e.g. organizational differentiation in some school subjects such as maths and foreign languages, further development of the curricula of individual school subjects, in particular the learning contents and procedures. The strategies of mastery learning as well as remediation in individual guidance have also been applied.

Research into cognitive styles is creating a basis for the provision of teaching materials that take the individual learner into account and teaching methods that serve various learner groups most efficiently.

2. Cognitive Profile consisting of various Cognitive Controls

Research has been concerned with cognitive controls for approximately thirty years. The following definition of a cognitive control is given by Messick (1970): Cognitive controls are information processing habits typical of an individual person, that is, his habits of observing, memorizing, dealing with problems, and solving them.

One possibility of analyzing variations of cognitive controls is to survey an individual as a processor of information. The various controls lay emphasis on different stages in information processing: observing, memorizing, thinking, problem solving, etc.

Recent research into the study of cognitive controls has been concerned with the establishment of a cognitive profile. In this study the components of the profile consist of seven different controls that have been selected from among some twenty controls studied. These seven controls describe the different stages of the information process in a way that ensures the minimum of overlapping. In addition, the measurement of these components is relatively simple. (Aimo-Viilo, 1983)

The seven cognitive controls selected to constitute the profile will be presented below.

1. Cognitive Complexity (K/Y)
This dimension describes the tendency of an individual to form a certain kind of inner representation of the outside world, especially of his social environment. This indicates the extent to which he observes his environment in a multi-dimensional process which discriminates stimuli. (Bieri et al. 1966)

2. Leveling - Sharpening (T/T)
This control describes the individual process of storing information in the memory. It also gives an account of the extent to which an individual is inclined to confuse similar information with that already existing in his memory. (Holzman-Klein, 1954)

3. Tolerance of Incongruous Experience (S/S)
This dimension describes the degree in which an individual is prepared to accept incongruent observations. (Klein et al. 1962)

4. Analytical - Global (A/K)
This control, which is related to the observation process, describes the degree in which an individual is inclined to perceive objects as separate from their background, and the degree in which he is capable of overlooking the confusing impact of the background. This control is also called Field-depedence - Field-independence. (Witkin et al. 1962)

5. Breadth of Category (K/L)
This control describes the breadth of the conceptual categories that the individual is inclined to form. (Pettigrew, 1958)

6. Focus - Non-focus (S/L)
This dimension describes the degree of extensive and intensive attention that the individual maintains. (Gardner-Long, 1962)

7. Reflective - Impulsive (H/I)
This control describes the speed at which an individual processes information and selects hypotheses and the degree in which he considers the various alternative solutions before replying to the question. (Kagan et al. 1964)

3. The Interrelation between Academic Achievement and the Cognitive Profile

In our study, which tested eighth-grade children at the upeer level of the Comprehensive School, we found significant differences between the subjects' cognitive profiles. However, specific types were discovered with very similar profiles of children belonging to a particular category (figure 1). (Aimo-Viilo, 1983)

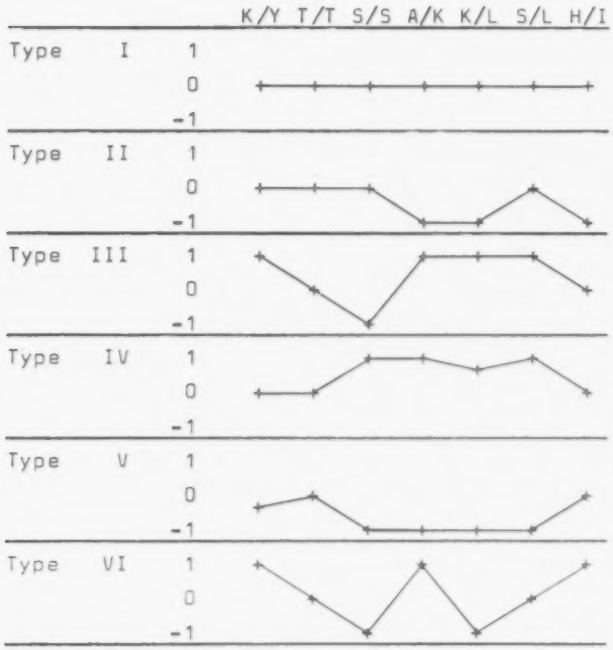

Figure 1. Median Profiles of the Types.

To establish the types the "Elementary Linkage Analysis" of McQuitty was applied (Cohen-Manion, 1982). This approach produced the following six types. The first to be distinguished from the experimental group was Type I, which is a typical middle group with a median profile

following the average of the profile scores of the components. The subjects of this type were average academic achievers as well as average achievers in more practical areas. The second type to be distinguished from the experimental group, Type II, consisted of children who were global and impulsive in approach and employed broad categories in defining things. In terms of achievement, this group was by far the lowest. The subjects of Type III were cognitively complex, intolerant of incongruous experience, analytical, employing narrow categories, and in their approaches focussing and selective. In their achievement they were the most successful. The children of Type IV were tolerant of incongruous experience, analytical, employing narrow categories, and focussing and selective in their approach. In their achievement this type was almost as successful as Type III. The median profiles of these two types resembled each other in their form. The subjects of Type V were cognitively simple, sharpeners, intolerant of incongruous experience, global, broad in their categorizing, non-focussing in their attention. In their achievement this type was to some extent superior to Type II. The remaining three subjects constituted Type VI. Their profiles were very similar to one another, even though one of the subjects was very different from others in his school achievement.

To find out the differences in academic achievement in Types I-VI, one-dimensional variance analysis was used. In the area of science, the differences between the various types were significant (at the level of 1 %); in the area of maths and all academic subjects (measured by the mean of report grades) there were almost significant differences (at the level of 5 %).

When comparing the means of the report grades gained by Types I-VI various school subjects we found that the children of Type III were the most successful.

According to their cognitive profiles the following traits characterise these children: cognitive complexity, analytical approach, narrow categorizing, and focussing and selective approach. The children of Type II were the least successful in all school subjects. Characteristic traits of Type II were global approach, broad categorizing and impulsiveness. The cognitive profiles of Types III and II differed greatly from each other.

The other types (I, IV, V, VI) also showed differences in academic achievement. When these types were ranked according to the means of the report grades in various school subjects or subject groups, the respective order of the types remained more or less stable.

Charles Letteri (1982) has carried out extensive research into the interrelations between the cognitive profile and academic achievement. He constructed the different types on the basis of their school achievement: superior academic achievers, average academic achievers, and those who fail to meet the grade-level requirement, and compared their profiles to one another. In our study we constructed the types on the basis of the similarity of profile and compared their academic achievement. In order to be able to compare our results to those gained by Letteri, we formed groups corresponding to those of Letteri on the basis of academic achievement.

The results of our study are parallel to those of Letteri's. The type of superior academic achievers differs from the corresponding type of Letteri in two components only: tolerance of incongruous experience, and breadth of category. As for breadth of category, the differences in the results may be due to differences in the cultural background that the measures display. Those who failed to gain the grade level distinctly showed similar results to Letteri, regarding the components analytical - global, and breadth of category (Leino-Leino, 1982).

4. The Application of the Results of the Profile Study to Teaching at School

The Spirit of the new Comprehensive School Act and Statutes requires the grouping of pupils to be based on heterogeneity when the new time resource quota system is introduced. On account of this any grouping based on the cognitive profile is unthinkable, since that would result in groups that are homogenous in academic achievement. It might be worthwhile to study the expediency of grouping based on some particular component; in that case the children's achievement would have improved due to teaching strategies applied to the specific group.

The question arises whether the cognitive profile of the teacher has any impact on the degree of variation he possesses when teaching different kinds of groups. According to our own experience the teacher may have regarded a group as a passive one, while in fact it has been taking its time and employing consideration. On the other hand, a group of children regarded by the teacher as noisy and difficult to deal with, may have been impulsive in character.

The different characteristics of these types should be discussed already in the course of teacher training. A would-be teacher ought to notice that in the teaching of different kinds of groups the available teaching methods must have a different kind of emphasis. If the teacher is aware of the fact that there are different pupils who learn in different ways and who differ in their cognitive profiles, it is possible for him to observe his group of students and adapt his strategies to their needs and select the teaching materials to suit them. Consequently, this fact has great significance also from the point of view of differentiation inside the group.

Remedial teaching has not produced as good results as was hoped. Teachers should realize that the teaching strategies they employ in the lessons must be changed in remediation work. The cognitive profile of the pupils in need of remediation does not allow them to profit from the strategies employed in the lessons.

Those who compile remediation materials should take into account the different types of cognitive profile and create remediation materials suited for their specific needs, materials that are individual, varied, and something out of the ordinary. The present remediation materials consist almost entirely of extra assignments for pupils.

To sum up our results: the fact that the level of pupil's academic achievement can be predicted from his cognitive profile gives ample cause to consider changing the cognitive profile in such a way that the level of achievement improves. It is a fact, too, that it will not be possible to determine everyone's cognitive profile, since the set of measuring devices is only available for research purposes and the actual measuring demands a great deal of work. Therefore, it is necessary to determine how the pupils whose profile should be changed ought to be selected. Another problematic question is to find the method changing the profile and the persons to participate in the process (for instance, the school psychologist, the school social worker, the vocational guidance teacher, the home room teacher, other teachers). Nevertheless, ethical viewpoints cannot be overlooked either. Can we change the pupil's cognitive profile without touching his personality, and to what extent should we accept changing one's personality?

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COGNITIVE STYLE IN SOLVING MATHEMATICAL PROBLEMS

Ritva Lapatto

1. Introduction

The study of various cognitive styles became popular especially in the United States in the 1970s but, as of late, also in Finland. It is true that research of this kind has been going on in psychology even for a longer period of time than cognitive psychology has gained ground, but in education this type of research is quite new. The concepts of cognitive psychology can easily be adapted into education as well, which has led to research on the correlation between instruction and learning.

In their report "Learning Difficulties in the Light of Cognitive Processes", 1982, Anna-Liisa and Jarkko Leino state that earlier studies on the differences between individuals could hardly be utilized in practice. Cognitive psychology has introduced a way to describe man's mental processes which can also be used in describing learning processes in school. At the same time, it gives us a means to avail ourselves of the results and testing methods of psychological research in order to overcome learning difficulties and to make teaching methods focus more on individuals.

The psychological study of cognitive processes, perception, learning, memorizing, thinking and decision making have - during the past few decades - changed our concept of man, who is now

seen as an active creature (von Wright, 1981). The reasons for the advance of cognitive psychology ever since the late 1950s are the information theory, cybernetics and game theory of the 1940s and the on-march of computers in the 1950s, says von Wright (1980).

The study of cognitive processes in the teaching of mathematics can be divided into two parts: (1) the study of memory and the structures of retained information and (2) the study of processing information (Leino, J., 1982).

2. Theoretical Basis

Human memory has both short-term and long-term capacities, and lately a lot has been written about this by both psychologists and educational scientists. In teaching mathematics, the limited nature of the short-term memory capacity makes it impossible for us to handle even one complex mathematical problem as a whole in terms of this memory (Leino, J., 1982).

Cognitive psychology aims at studying the general laws of information processing and the holistic nature of the processes. In pedagogy and teaching, the study on information processing has particularly focused on individual differences. Speed is a factor which greatly varies from one individual to another. It does not mean, however, that a fast student would learn better than a slow one but it merely is a different way of learning, which places the teacher in a difficult position, since he is supposed to diversify his teaching to satisfy the needs of both types.

In teaching mathematics one can observe how even the simplest problems are solved by the students in different ways. The different means to solve mathematical problems are called problem solving strategies and the means by which students glean new knowledge and skills are called learning strategies. The concept of a strategy is generally understood as goal-oriented. As an individual grows and develops he forms different strategies

and the teacher can influence the coming about of these (Leino, J., 1982). When a strategy turns into a permanent feature in the learning process, we talk about a cognitive style, and since we are dealing with learning, we can use the term learning style.

Cognitive styles are a fairly new area of study, which means that some of the definitions and theories are not yet fully defined. Messick (1976) defines the cognitive style as a way typical of an individual to observe, remember, think and solve problems or, in other words, a way characteristic of him to process information. In cognitive psychology studies learning strategies are also characteristic behavior patterns of this type (von Wright et al., 1979), but the difference is that in psychology one looks for general laws, whereas in cognitive style studies one focuses on individual differences (Laine, 1981).

The cognitive style is regarded as something without values in contrast to intelligence and it is influenced by affective and emotional factors as well (Harvey et al., 1961; Sigel et al., 1974). The cognitive style is taken to be relatively permanent but not entirely unchangeable (Sigel et al., 1974; Harvey et al., 1961; Entwistle, 1981). Styles also differ from capabilities in that the former are bipolar, hovering anywhere between two extremes (which are opposites of each other), of which either can be advantageous depending on the situation, whereas the latter are unipolar and vary from very small to very great.

Some of the most important dimensions of the cognitive style are (Kagan et al., 1963; Sigel et al., 1974; Pask, 1976; Messer, 1976; Messick, 1976; Leino, A-L., 1980; Leino, J., 1981, 1982; Letteri, 1980 and Laine, 1981):

1. Conceptual complexity, which refers to the differences how individuals interpret the world, especially the world of social behavior, in a multidimensional and distinguishing way.
2. Levelling, - sharpening, which refers to the extent of mixing new stimuli with older information already retained in memory.
3. Enduring contradiction refers to differential ability to accept observations which are unconventional and contradict traditional experiences.

4. Breadth of categorization refers to that consistency with which an individual engaged in categorization problems defines the acceptable boundaries of the categories.
5. Focusing - scanning is a dimension, which refers to individual differences in the extent and intensity of the use of attentiviness.
6. Field-independence - field-dependence have to do with the observation stage. This is one of the styles under study and in this study the field-independence style is referred to as analytical and the field-independence as holistic. An analytical person is able to make analytical observations. He can find details or a hidden figure in an interfering background. A holistic person is mainly capable of seeing the the total picture.
7. Reflection - impulsivity is likewise under study. A reflective person attempts to ponder all alternatives, before he commences solving the problem or giving an answer to a specific question. An impulsive person, on the other hand, launches immediately on solving the problem or gives the first answer that comes to his mind, which he finds necessary to correct afterwards.

3. Conducting the study

The subjects in the study were sixth-graders (12-13-year-olds) in the comprehensive school because we wanted to see if the concept of style would offer help in grouping the students in the upper level of the comprehensive school after streaming was phased out. The students in the study attended sixth grades in Helsingin Yliopiston Harjoittelukoulu II, in Munkkivuoren ala-aste and in Haagan ala-aste - a total of 147 students.

The analytical - holistic traits of the subjects were examined with the group version of Witkin's hidden figures or the GEF-test. Each student received a booklet, which contained simple geometric forms hidden in complex patterns and which became more and more difficult as one advanced from one picture to another. The student had to find that simple form in a given period of time and

indicate it by tracing it in pencil. A high score indicates that the student is analytical, a low score that he is holistic.

The reflective - impulsive traits were examined with Kagan's Matching Familiar Figures Test or the MFF-test. It is an individual test in which, among six similar pictures, the student has to find the one which matches exactly with the model. There were a total of twelve picture series in the test. The student was timed as he went through each picture series for giving his first answer and likewise the number of wrong answers was noted. He had a chance to correct his answer till he found the right picture. Purely reflective persons were those who took more time to thinking out the answer and whose first answer was in most cases correct. Impulsive students answered very quickly and thought they were giving the right answer but actually had given the wrong one.

Problems in the study:

1. Do the analytical - holistic characteristics have anything to do with how well a student is doing in school?
2. How do the students from the two extremes of this style differ in solving mathematical problems?
3. Do the reflective - impulsive characteristics have anything to do with how well a student is doing in school?
4. How do the students with equal abilities from the two extremes of this style differ in solving mathematical mltiple choice problems?
5. Are there any differences in the way the reflective and impulsive students respond to the methods used in the study?

There were three mathematical measuring instruments. One was a multiple choice test which comprised of twelve problems each with four alternatives (with 0-4 possible right answers). The student was asked to write TRUE or FALSE next to each choice. The problems were easy but the time was limited so that we could find out whether the student would have the patience to think through all the alternatives. The second test comprised of seven verbal

mathematical problems. Some of them had extra information and the geometry problems could be solved in more than one way. The idea was to see if students of different types would solve the problems in different ways. The third test was the national mathematical test of the League of Finnish Mathematics and Sciences Teachers. We furthermore received the grades for mathematics, reading, writing and a foreign language for each student.

4. Results

The students were first divided into analytical and holistic ones and into reflective and impulsive ones on the basis of the averages in the psychological test. In calculating the correlation between the test results and grades at school we arrived at the following:

	GEF-test	MFF-test
Mathematics	0.38	0.20
Reading	0.26	0.14
Writing	0.30	0.07
Foreign Language	0.20	0.06

The correlations are relatively low when one considers that the correlation between mathematics and reading was 0.76. The low correlation was desirable in the sense that when the tests are used as a basis for grouping the students, each group would thus become heterogeneous as far as school grades are concerned. In the present study purely analytical students were those who differ from the average by the standard deviation. The average in the GEF-test was 9.5 and the standard deviation 5.1, which meant that those with a score of 15-18 (the test maximum was 18) were analytical. There was a total of 31 of them. Those with a score of 0-4 were holistic. There was a total of 28 of them. In calculating the correlation of the FEF-test and mathematics grades of these 59 students 0.60 is obtained. A holistic student faces therefore difficulties in school mathematics more often than an analytical student.

Furthermore, we could find out that the analytical students can better solve verbal mathematical problems than the holistic students and they do not as easily fall victims for the additional information in the problem. In addition, they are able to find a shortcut in solving geometry problems more often that the holistic students do. The reflective - impulsive traits produced even fewer differences in how well the students can solve mathematical problems. We were able to obtain 35 purely reflective students and 43 purely impulsive students. The correlation between the MFF-test and the results of the national mathematics test was 0.34 for this group in comparison with 0.22 for the entire number of students participating in the study. As we tried to find out the reasons for the increased correlation for these pure types, it became clear that the national mathematics test mentioned above had a part in which there were twenty transformation problems wherein the impulsive students clearly scored less. This could be explained by saying that impulsive students do not have the patience to concentrate on monotonous problems of the same type.

We would detect no connection between the results of the multiple choice test and the school grades in mathematics.

Since neither of the styles under study divided the students as far as their school grades were concerned, we felt it was necessery to try to investigate the students' preference for various methods of teaching and lesson materials. A study like this is best done with one's own class and that is why one sixth grade and one seventh grade in Helsingin Suomalainen Yhteiskoulu were tested with the reflectiveness - impulsiveness test. This test was chosen since it seemed to distinguish between different types of students in a normal class.

The students' preference for two completely opposite teaching methods was investigated by teaching them first with a teacher-centered method and then by having them work independently. The tests were carried out in the so-called group hours which means that the class is divided into two groups of 16 students during

their mathematical lesson. The grouping is done alphabetically and not by any other special method. The grouping is possible because of the time resource quota system being carried out in the school. The same teacher teaches both halves of the class because one of the groups is having some other subject when the other is having mathematics.

Later on the students were asked which of the methods used they preferred. The results were correlated to pure types and they were surprisingly clear indeed. The reflective students preferred teacher-centered teaching and the impulsive students in general preferred working independently. This led us to think that school work should be organized according to the preference of the students, if the students were grouped according to the results of this test. The results in evaluating lesson materials coincided with the results from the teaching method inquiry. The students were given a chance to choose between mechanical problems and verbal problems about the same topic. The reflective students had no problem preferences, whereas the impulsive students prefeffed verbal problems.

5. Discussion

In the present study we concentrated only on two cognitive styles, which were chosen because earlier studies, mainly in the United States, have shown that analytical-holistic traits have some connection with learning mathematics and the methods used in teaching. That study dealt with the deductive and inductive teaching methods.

The results of the study were pleasing in the sense that they do not separate good students from the bad in mathematics, nor in Finnish nor in a foreign language. This is the reason why one seventh grade class in Helsingin Suomalainen Yhteiskoulu was divided into groups with the analytical - holistic test and another with the reflective - impulsive test in the school year 1983-1984. The groups remain the same in all the subjects in

which the time resource quota system is being used. We are trying to find out whether either test is suitable for grouping students in this manner.

One disadvantage of grouping students like this is the fact that testing is ardous and time consuming. It could well be that the teachers - once they are used to this new way of thinking in observing their students - are able to group them without tests. We can use the students themselves by describing a reflective person and an impulsive person to them and then by asking which they think they are. All of this must be done without showing any preference to either of the types.

In the study we found out in examining the differences between the classes that the teacher could have had some influence on the class since a holistic student had become a bit more analytical. There are no facts in support of this supposition which would deserve study, though. The styles are regarded as quite permanent but long range influence can have a bearing.

It should furthermore be investigated whether the teacher's influence perhaps is unintentional. Perhaps the teacher is purely one type and hence his influence catches on in the students as they indentify themselves with the teacher. On the other hand, the teacher's influence can be deliberate. In the course of years he has found ways for the students to learn solve geometry problems, for instance, and he wants deliberately to avail himself of them. Perhaps he prefers one teaching method to another and he could very well always choose certain type of lesson material over some other type.

The cognitive styles are very important in understanding and utilizing the interaction between teaching and learning processes. The teacher can benefit from knowing as much as possible about his students, and his teaching job becomes more interesting and rewarding when he at least knows that the students can be different in this way, too.

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UNTERSUCHUNG VON EINMALEINSKENNTNISSEN IM MATHEMATIKUNTERRICHT IN DER BUNDESPEPUBLIK DEUTSCHLAND

Gustav Adolf Lörcher

1. Einleitung

Über welches Wissen in Mathematik sollte jeder Schüler sicher verfügen, wenn er die Schule verlässt?

Bei der Beantwortung dieser Frage gibt es grosse Meinungsunterschiede. Während Industrie und Handwerk bei der Auswahl jugendlicher Bewerber um einen Ausbildungsplatz nach wie vor z.B. schriftliche Rechenverfahren und Bruchrechnen zu den unverzichtbaren Bestandteilen zählen und darauf in betrieblichen Eignungstests grossen Wert legen (Lörcher 1980), sieht die Realität in Alltag und Berufswelt heute in Bezug auf diese beiden Fertigkeiten grossenteils schon anders aus: schwierigere Rechnungen werden nicht mehr schriftlich, sondern mit einem Rechner ausgeführt.

Bei allen Meinungsunterschieden ist aber unumstritten, dass jeder Erwachsene auch künftig über einige sichere Grundfertigkeiten im Kopfrechnen verfügen sollte, da er nur so ein von Rechnern unabhängiges individuelles Kontrollsystem aufbauen kann, das ihn befähigt, die Plausibilität von Rechenergebnissen zu beurteilen und grosse Fehler zu erkennen.

Einen zentralen Platz bei diesen Grundfertigkeiten nimmt die sichere Beherrschung des Einmaleins ein. Es wird nicht nur später bei Kopfrechnen und Überschlagsrechnung benötigt, sondern nimmt auch im Mathematikunterricht eine Schlüsselstellung ein. Viele komplexere Verfahren, die der Schüler in den späteren Schuljahren zu lernen hat, setzen voraus, dass er über sichere Einmaleinskenntnisse verfügt: das gilt für schriftliche Multiplikation und Division, aber auch für die Lösung von Sachaufgaben, für geometrische Berechnungen und für die meisten algebraischen Verfahren. Ein Schüler, der Lücken im Einmaleins hat, leidet unter laufenden Misserfolgen bei der Durchführung dieser Verfahren, auch wenn er die Verfahren als solche verstanden hat.

Was heisst nun Beherrschung des Einmaleins? Dazu gehört einmal Rechensicherheit, zum andern eine gewisse Schnelligkeit. Versucht man abzuschätzen, wie gross die Rechensicherheit höchstens sein kann und wie gross sie mindestens sein sollte, so muss man zunächst davon ausgehen, dass es ein unerreichbares Wunschziel ist, bei allen Schülern eine 100%-ige Rechensicherheit zu erreichen. Eine kürzliche Untersuchung bei Mathematikstudenten und Mathematikreferendaren hat gezeigt, dass selbst sie im Durchschnitt nur 99% aller Einmaleinsaufgaben lösten. Eine im Normalunterricht der Schule erreichbare Obergrenze dürfte also eher unter diesem Wert liegen.

Auf der anderen Seite kann man ausrechnen, wie gross die Erfolgswahrscheinlichkeit für das richtige Ergebnis bei der schriftlichen Multiplikation einer 3-stelligen mit einer 4-stelligen Zahl höchstens ist, wenn ein Schüler im Durchschnitt 95% aller Einmaleinsaufgaben richtig löst, und wenn man annimmt, dass die Erfolgswahrscheinlichkeiten bei den Multiplikationen im Kopf unabhängig voneinander sind. In diesem Fall würde der Schüler eine solche Multiplikation nur mit $0{,}95^{12} = 54\%$ Wahrscheinlichkeit richtig lösen, wobei noch vorausgesetzt wurde, dass er das Verfahren sicher

beherrscht und keine Fehler bei der Addition macht. Wollte er bei einer derartigen schriftlichen Multiplikationsaufgabe eine 75%-ige Lösungswahrscheinlichkeit erreichen, so müsste er das 1x1 zu mindestens 97,5% beherrschen (vgl. dazu auch Brousseau 1974).

Neben der Rechensicherheit spielt auch die Schnelligkeit eine beträchtliche Rolle. Dauert es zu lange, bis der Schüler die Antwort auf eine Einmaleinsaufgabe gefunden hat, so ist die zeitliche Kapazität seines Arbeitsspeichers im Gedächtnis erschöpft und er hat die vorausgegangenen Überlegungen vergessen. Da auch die Informationskapazität dieses Arbeitsspeichers eng begrenzt ist (nur ungefähr 5 Gedanken können gleichzeitig festgehalten werden), sollte der Schüler möglichst viele der Einmaleinskombinationen im Langzeitgedächtnis gespeichert haben, um sich den Arbeitsspeicher nicht mit der Herleitung einer Einmaleinskombination zu blockieren (Kilborn 1979). Ziel sollte also eine möglichst vollständige Automatisierung des 1x1 sein, so dass der Abruf der einzelnen Kombinationen weniger als 5 Sekunden benötigt.

Versucht man sich darüber zu informieren, wie es in der Realität des Mathematikunterrichts mit der Rechensicherheit beim Einmaleins aussieht und wie lange die Schüler im Durchschnitt für einzelne Kombinationen benötigen, so fällt auf, dass es nur relativ wenige Untersuchungen aus den letzten Jahren zu diesen Fragen gibt. Eine dieser wenigen Untersuchungen stammt aus Dänemark (Nielsen 1977). Dort wurde mit Hilfe des probabilistischen Messmodells von G. Rasch auf Grund einer 1974/75 bei insgesamt 1418 Schülern im 2. bis 6. Schuljahr vorgenommenen Untersuchung auf einer Rationalskala geschätzt, wie leicht die einzelnen Einmaleinskombinationen sind.

Dabei wurden folgende Werte ermittelt:

TABELLE 1. Leichtigkeitsschätzungen der 1x1-Kombinationen

	0	1	2	3	4	5	6	7	8	9	10
0	9,4	0,72	0,98	0,75	0,69	0,72	0,64	0,50	0,76	0,66	0,74
1	0,63	0,43	1,8	3,1	2,9	2,6	3,1	2,8	3,6	4,7	3,9
2	0,78	2,9	1,2	2,0	2,2	5,2	2,3	2,1	1,7	3,3	6,0
3	0,57	1,9	2,1	2,4	1,2	2,3	1,1	2,3	0,90	0,87	3,1
4	0,39	1,7	1,4	1,6	0,90	2,2	1,1	0,84	0,46	0,41	3,9
5	0,65	2,0	3,8	3,4	1,6	1,1	1,1	1,0	1,1	0,99	3,2
6	0,44	2,4	1,8	1,0	1,2	1,5	0,35	0,27	0,18	0,28	2,6
7	0,56	2,2	2,6	1,5	0,83	0,99	0,21	0,20	0,13	0,20	2,7
8	0,87	2,3	2,0	0,84	0,60	1,2	0,20	0,12	0,15	0,24	4,8
9	0,76	3,7	3,0	0,74	0,52	1,4	0,25	0,19	0,23	0,26	3,0
10	0,46	2,0	2,7	3,3	3,5	2,9	4,3	3,8	4,6	2,4	1,1

Die leichteste Kombination war demnach 0x0. Sie war 9,4 mal so leicht wie die Kombinationen 6x3 und 5x7, die mit dem Wert 1,0 eine durchschnittliche Leichtigkeit besassen; 4 mal so schwer wie diese Kombinationen war 9x6 mit der Leichtigkeit 0,25, das seinerseits rund halb so schwer war wie die schwierigste Kombination 8x7.

In einer etwas älteren schwedischen Untersuchung (Olofsson 1973) mit 383 Schülern der Schuljahre 7-9 wurde ein durchschnittlicher Fehlerprozentsatz von 2% pro Schüler ermittelt, wobei jeder Schüler durchschnittlich 360± 160 Sekunden für die dort zugrundegelegten 100 1x1-Kombinationen benötigte. In der gleichen Untersuchung sind auch Angaben zur Umkehrung des Einmaleins sowie zu den einzelnen aufgetretenen Fehlern enthalten.

Interessante Angaben zu Einmaleinskenntnissen im 4. bis 7. Schuljahr finden sich in den Teilrapporten "Multiplikation" des Forsøg med regnemaskiner i undervisningen: 3.-7. klasse 1975-1981 von J.Ethelberg-Laursén in Århus (Ethelberg-Laursén 1982).

Hier fiel der Fehlerprozentsatz vom 3. bis 7. Schuljahr in den Kontrollklassen von rund 16% auf 6%, in den Versuchsklassen dagegen, in denen die Multiplikation mit Hilfe des Taschenrechners eingeführt worden war, von 8% auf 3% und war damit - im Widerspruch zu einem weit verbreiteten Vorurteil - jeweils nur halb so gross wie in den Klassen, die das Einmaleins ohne Hilfe des Taschenrechners gelernt hatten. In allen diesen Untersuchungen trat ein Phänomen zutage, auf das auch schon in früheren Untersuchungen hingewiesen worden war: die hohe Fehleranfälligkeit der Kombinationen mit Null (Prien 1969, Schonell&Schonell 1965).

Im folgenden soll über einige eigene Untersuchungen aus den letzten Jahren zu Einmaleinskenntnissen berichtet werden. Zunächst wird untersucht, welchen Erfolg gezielte Kurzübungen beim Einüben des Einmaleins im 3. Schuljahr haben können. Dann werden einige Daten zu Einmaleinskenntnissen von Schülern in der Hauptschule (5. bis 9. Schuljahr; Hälfte des Altersjahrgangs, unterer Leistungsbereich) und in der Realschule (5. bis 10. Schuljahr; ungefähr 1/4 des Altersjahrgangs; mittlerer bis oberer Leistungsbereich) analysiert. Die Daten stammen aus Examensarbeiten junger Lehrer sowie aus eigenen Auswertungen.

2. Einmaleinskenntnisse im 3. Schuljahr

Untersuchungsmethode

Nach dem in Baden-Württemberg gültigen Lehrplan sollen die Schüler während des 3. Schuljahrs im Mathematikunterricht sichere Kenntnisse im Einmaleins erwerben. Um diese Phase möglichst rationell und möglichst anregend für die Kinder zu gestalten, wurde während der Jahre 1978 - 1981 in 10 Klassen folgender Versuch gemacht: zunächst wurden in einem Vortest zum 1x1 alle 121 Kombinationen von 0x0 bis 10x10 in zufälliger Reihenfolge abgefragt. Danach wurden während mehrerer Wochen in 5- bis 15-minütigen Kurzübungen jeweils

zu Beginn der Mathematikstunde zunächst die einzelnen Reihen getrennt, dann gemischt eingeübt. Dabei wurden Spiele (Würfelspiele, Domino, Ballspiele mit Zuruf von Aufgaben) und motivierende Übungsformen mit Selbstkontrolle (Puzzles; Punktebilder; Aufgaben mit Geheimschrift, die bei richtiger Lösung ein Lösungswort ergaben, eingesetzt. Bei der Auswahl der Zahlen wurde besonderes Augenmerk auf die im Vortest als schwierig erkannten Kombinationen gelegt.

Der Unterricht wurde von Junglehrerinnen in der 2. Ausbildungsphase, die in diesen Klassen eigenständig unterrichteten, durchgeführt und in ihrer Examensarbeit zur 2. Dienstprüfung beschrieben und ausgewertet.

Nach Beendigung der Übungsphase (jeweils noch in der 1. Hälfte des 3. Schuljahrs) wurde der gleiche Test nochmals als Schlusstest durchgeführt.

Untersuchungsergebnisse

Bei den untersuchten 244 Schülern in den 10 Klassen fiel der durchschnittliche Fehlerprozentsatz pro Schüler von 16,0% ± 14,7% vor Beginn der Übungsphase auf 4,9% ± 7,9% im Nachtest. Der Anteil der Schüler, die 0 Fehler hatten, stieg von 5% auf 20%, der Anteil derer, die höchstens 5% Fehler hatten, stieg von 1/3 auf rund 70% und der Anteil derer, die mehr als 10% falsch hatten, sank von der Hälfte auf 20% der gesamten Schülerzahl.

Während vor Beginn der Übungsphase der Zeitbedarf umgerechnet pro Aufgabe 14,0 Sekunden ± 8,4 Sekunden bei den Schülern betrug, verringerte er sich auf 9,2 ± 5,0 Sekunden beim Schlusstest. Die Korrelation von Fehlerzahl und Zeitbedarf lag zunächst bei 0,38 und stieg dann auf 0,62.

Die Fehlerprozentsätze bei den einzelnen Kombinationen veränderten sich wie folgt:

TABELLE 2. Fehlerprozentsätze bei 1x1-Kombinationen im 3. Schuljahr

Vor Einübung

	0	1	2	3	4	5	6	7	8	9	10	ø
0	1	28	37	36	36	36	38	41	37	39	35	33
1	30	7	2	0	2	2	2	3	1	4	4	5
2	34	6	6	5	3	2	4	6	4	5	0	7
3	32	5	5	6	7	2	11	8	16	15	2	10
4	34	6	4	7	11	3	13	16	23	23	2	13
5	36	3	4	3	8	10	11	14	15	17	4	11
6	31	7	6	6	17	8	24	34	43	34	2	19
7	37	3	6	11	14	9	37	41	47	37	4	22
8	30	4	7	17	21	10	45	51	44	32	2	24
9	31	2	7	16	23	12	35	39	37	20	2	21
10	36	5	3	4	3	4	4	4	3	3	6	7
ø	30	7	8	10	13	9	20	23	25	21	6	16

Nach Einübung

	0	1	2	3	4	5	6	7	8	9	10	ø
0		5	5	5	4	5	5	6	4	7	5	5
1	5	2	1	1	0	1	1	0	0	1	2	1
2	3	2	2	2	0	2	1	2	2	2	0	2
3	4	2	2	5	4	1	4	5	4	5	1	3
4	4	2	0	5	6	1	9	3	7	12	1	5
5	4	1	2	1	3	4	5	7	3	5	1	3
6	5	3	1	4	9	2	7	13	18	14	1	7
7	5	2	2	5	9	2	21	16	25	20	0	10
8	5	4	4	7	7	2	22	24	18	16	1	10
9	6	2	2	7	12	4	17	15	16	8	1	8
10	7	2	0	0	1	1	1	1	2	1	3	2
ø	4	3	2	4	5	2	8	8	9	8	1	5

Fasst man Kombinationen ähnlicher Schwierigkeit zusammen, so kann man die ganze 1x1-Tabelle in 5 Gebiete einteilen:

TABELLE 3. Fehlerprozentsätze in den 5 Teilgebieten

Vor der Übungsphase

Kombinationen	Anzahl	Fehlerprozent	Schwerste	Leichteste
-mit Null	20	34,7%±3,3%	0·7 41,4%	0·1 27,9%
-mit 1,2,10;0x0	52	3,8%±1,8%	9·2 7,4%	2·10 0,4%
-niedere(3x3 bis 5x5)	9	6,3%±3,0%	4·4 11,1%	3·5 2,0%
-gemischte(3x6 bis 9x5)	24	14,4%±4,8%	4·8 23,4%	6·3 5,7%
-hohe(6x6 bis 9x9)	16	37,5%±7,8%	8·7 51,2%	9·9 20,0%
	121	16,0%	8·7 51,2%	2·10 0,4%

Nach der Übungsphase

Kombinationen	Anzahl	Fehlerprozent	Schwerste	Leichteste
-mit Null	20	5,1%±1,0%	10·0 7,4%	2·0 3,3%
-mit 1,2,10;0x0	52	1,4%±0,9%	7·2,8·2 4,1%	0·0 0,0%
-niedere(3x3 bis 5x5)	9	3,3%±1,8%	4·4 5,7%	3·5 0,8%
-gemischte(3x6 bis 9x5)	24	5,8%±2,8%	4·9 12,3%	7·5 1,6%
-hohe(6x6 bis 9x9)	16	16,9%±4,9%	7·8 24,6%	6·6 6,6%
	121	4,9%	7·8 24,6%	0·0,1·7 1·8,2·4 0,0% 2·10

Die Angabe z.B. bei den Nullkombinationen nach der Übungsphase 5,1% ± 1,0% bedeutet, dass 5% aller 20 Nullenkombinationen (von 1x0 bis 10x0 und 0x1 bis 0x10) falsch gelöst wurden bei einer Standardabweichung von 1% über alle Nullenkombinationen.

Diskussion der Ergebnisse

Auffallend ist insgesamt die starke Reduzierung der Fehlerzahl um mehr als 2/3. Regelmässige Kurzübungen scheinen demnach nicht nur ein rationelles, da wenig zeitaufwendiges, sondern auch ein äusserst effektives Mittel zur Sicherung von Lernergebnissen zu sein. Besonders stark haben gerade die schwächeren Schüler davon profitiert.

Der durchschnittliche Zeitbedarf pro Aufgabe konnte zwar ebenfalls deutlich (um gut 1/3) reduziert werden, wenn auch etwas weniger ins Auge fallend. Um eine Automatisierung bei den Schülern erreichen zu können, scheint der Übungszeitraum zu kurz zu sein. Dafür wären eher langfristig angelegte Übungen notwendig. Der deutliche Anstieg der Korrelation zwischen Fehlerzahl und benötigter Zeit dürfte bedeuten, dass eine Automatisierung am ehesten bei den rechensicheren Schülern eingesetzt hat.

Untersucht man, welcher Lernerfolg bei den einzelnen Kombinationen eingetreten ist, so zeigt sich, dass er bei den Kombinationen mit Null am grössten war. Ihr Fehleranteil verringerte sich um nahezu 6/7. Dieses Ergebnis ist deshalb so beachtenswert und von grosser didaktischer Relevanz, weil es zeigt, dass eine hohe Fehleranfälligkeit bei Nullenkombinationen nicht unvermeidlich ist, sondern sich durch gezielte Beachtung in den Übungen weitgehend abbauen lässt.

Überdurchschnittlich stark war der Lernerfolg ausserdem vor allem bei den leichteren Kombinationen (2-er-Reihe, 5-er-Reihe, 10-er-Reihe, Multiplikation mit 1 und mit 2).

Am resistentesten gegen gezielte Übungen scheinen die schwierigen hohen Kombinationen (wie 7x8, 7x9, 8x7) zu sein. Immerhin gelang auch hier eine Reduzierung der Fehlerzahl um rund 50%. Um eine zureichende Sicherheit bei diesen Kombinationen zu erreichen, scheinen jedoch längerfristige Übungen erforderlich.

Vergleicht man den durchschnittlichen Fehlerprozentsatz in den 5 Gebieten ähnlicher Schwierigkeit, so zeigt sich - ausser bei den Nullkombinationen - vor und nach der Übungsphase ein ähnliches Bild: der Fehlerprozentsatz verdoppelt sich jeweils ungefähr von den Kombinationen mit 1, 2 und 10 über die niederen Kombinationen (3x3 bis 5x5) und die gemischten Kombinationen bis hin zu den hohen Kombinationen (6x6 bis 9x9), wobei der Schwierigkeitsanstieg

von den gemischten zu den hohen Kombinationen etwas stärker als zwischen den anderen Gebieten ist. Damit kommt man zu einer ähnlichen Schwierigkeitsverteilung wie sie auch in der Untersuchung von Nielsen (Nielsen 1977) zutage trat.

Unabhängig von den Übungen scheint es also sehr leichte, leichte, mittlere und schwere Kombinationen zu geben, die alle durch Übungen besser gesichert werden können, deren Verhältnis untereinander sich dadurch aber kaum ändert.

Um durch längerfristige Übungen am ehesten eine Reduzierung des Fehlerprozentsatzes erreichen zu können, würde es sich also lohnen, neben gezielten Übungen zur Multiplikation mit Null sich vor allem auf die 16 hohen Kombinationen von 6x6 bis 9x9 zu konzentrieren.

3. Einmaleinskenntnisse in der Sekundarstufe

Untersuchungsmethode

Die folgenden Testergebnisse stammen aus 33 Klassen des 5. bis 10. Schuljahrs (n=779), die nicht zufällig ausgewählt wurden. Die Testergebnisse können demnach nur in Groben Anhaltspunkte dafür liefern, wie die realen Verhältnisse im Mathematikunterricht aussehen.

Der Untersuchungszeitraum in den Hauptschulen (Hälfte des Altersjahrgangs, mittlerer bis unterer Leistungsbereich; entspricht der früheren kansakoulu) war 1981 und 1982, der in den Realschulen (entspricht der früheren oppikoulu) 1982 und 1983.

Die Klassen und Schülerzahlen verteilten sich wie folgt auf die einzelnen Schularten und Schuljahre:

Verteilung der Klassen

auf Schul-arten	auf Schuljahre 5.	6.	7.	8.	9.	10.	
Hauptschule	2	5	4	5	3	-	19
Realschule	1	4	2	3	3	1	14
Summe	3	9	6	8	6	1	33

Verteilung der Schüler

auf Schul-arten	auf Schuljahre 5.	6.	7.	8.	9.	10.	
Hauptschule	55	128	102	110	54	-	449
Realschule	25	97	46	70	76	16	330
Summe	80	225	148	180	130	16	779

Es wurden 2 Testformen benützt. Bei beiden standen alle 121 Kombinationen jeweils auf einem Blatt in zufälliger Reihenfolge. Die Schüler arbeiteten ohne Zeitbeschränkung. In keiner der Klassen waren vor dem Test gezielte Einmaleinsübungen gemacht worden.

Untersuchungsergebnisse

Der durchschnittliche Fehlerprozentsatz pro Schüler lag bei 5,1% ± 6,7%. In den einzelnen Altersstufen und Schularten sahen die Ergebnisse wie folgt aus:

TABELLE 4. Fehlerprozentsätze in Schularten und Schuljahren

Schulart	Schuljahre 5./6.	7./8.	9./10.	insgesamt
Hauptschule	6,1 ± 8,7	4,9 ± 5,5	2,8 ± 4,0	5,1 ± 7,4
Realschule	5,3 ± 5,5	5,2 ± 6,2	4,1 ± 4,0	5,0 ± 5,4
Insgesamt	5,8 ± 7,6	5,0 ± 5,8	3,6 ± 4,1	5,1 ± 6,7

Während die Unterschiede zwischen den Schularten gering bleiben, ist vor allem in der Hauptschule eine deutliche Verringerung des Fehlerprozentsatzes mit zunehmendem Schuljahr festzustellen.

Untersucht man die Fehlerverteilung auf die einzelnen Schüler, so zeigt sich, dass rund 1/6 der Schüler fehlerlos arbeitete (Hauptschule 20%, Realschule 14%). Bei ungefähr der Hälfte der Schüler lag der Fehlerprozentsatz unter 2%, bei rund 2/3 unter 5%. Ungefähr 1/6 der Schüler hatte mehr als 10% der Einmaleinskombinationen falsch. Es war eine deutliche Fehlerkonzentration bei einer Minderzahl von Schülern festzustellen: Ungefähr 50% aller aufgetretenen Fehler wurden von nur 14% der Schüler gemacht. Auf der anderen Seite entfielen auf 50% der Schüler nur knapp 10% aller gemachten Fehler.

Rechnet man den Zeitbedarf für den ganzen Test auf die durchschnittliche Einzelaufgabe um, so erhält man für die Schularten und die Altersstufen folgende Ergebnisse:

TABELLE 5. Zeitbedarf durchschnittlich pro 1x1-Kombination (in s)

Schulart	Schuljahre 5./6.	7./8.	9./10.	insgesamt
Hauptschule	4,7 ± 1,9	3,7 ± 1,3	3,1 ± 1,3	4,0 ± 1,7
Realschule	3,5 ± 1,3	2,7 ± 0,8	2,6 ± 0,6	3,0 ± 1,0
Insgesamt	4,2 ± 1,8	3,3 ± 1,2	2,8 ± 1,0	3,6 ± 1,5

Die Korrelation zwischen Fehlerzahl und Zeitbedarf lag jeweils in der Nähe von 0,2.

Die Fehlerprozentsätze bei den einzelnen 1x1-Kombinationen lagen in beiden Schularten ähnlich und sollen deshalb zusammengefasst dargestellt werden:

TABELLE 6. Fehlerprozentsätze bei den einzelnen 1x1-Kombinationen

	0	1	2	3	4	5	6	7	8	9	10	ø
0	0	16	12	16	14	14	15	15	15	15	11	13
1	17	1	1	0	1	0	1	1	0	1	1	2
2	17	2	1	1	1	1	1	1	1	1	1	2
3	17	2	1	2	1	1	3	2	3	4	1	3
4	18	1	1	2	5	1	4	4	7	5	1	4
5	16	2	1	1	1	1	3	3	4	4	1	3
6	15	1	3	1	3	3	3	7	11	9	1	5
7	18	1	1	2	4	3	8	9	14	9	1	6
8	16	1	2	3	10	2	12	13	12	9	0	7
9	16	1	1	3	6	4	10	8	10	5	1	6
10	18	1	1	1	2	1	1	1	1	1	2	3
ø	15	3	2	3	4	3	5	6	7	6	2	5

Fasst man jeweils wieder 1x1-Kombinationen ähnlicher Schwierigkeit zusammen, so erhält man für die 5 Teilgebiete folgende Verteilung:

TABELLE 7. Fehlerprozentsätze in den 5 Teilgebieten

Kombinationen	Anzahl	Fehlerprozent	Schwerste		Leichteste	
-mit Null	20	15%±2%	7·0	18%	0·10	1%
-mit 1,2,10;0x0	52	1%±0,5%	6·2	3%	0-0,8-10	0,1%
-niedere (3x3 bis 5x5)	9	2%±1%	4·4	5%	3·4	1%
-gemischte (3x6 bis 9x5)	23	4%±2%	8·4	10%	6·3	1%
-hohe(6x6 bis 9x9)	16	9%±3%	7·8	14%	6·6	3%
	121	5,1%	7·0	18%	0-0,8-10	0,1%

Auffallend ist hier die hohe Fehlerkonzentration bei der Multiplikation mit Null. Etwas mehr als 50% aller im 1x1-Test aufgetretenen Fehler waren Fehler bei Multiplikation mit Null. Dieser Anteil lag in beiden Schularten ungefähr gleich hoch.

Bei den Nullenfehlern war auffallend, dass sie bei einzelnen Schülern systematisch auftraten. Während ungefähr 50% aller Schüler überhaubt keine und weitere 30% nur sporadisch Nullenfehler machten, machten gut 10% systematisch in einer Richtung Nullenfehler (entweder a·0=a oder 0·a=a, wobei - im Unterschied zur Grundschule - der 1. Fall etwas häufiger auftrat) und weitere 10% systematisch in beiden Richtungen (d.h. sie hatten jeweils mindestens die Hälfte aller Nullenkombinationen falsch). Auf die 20% der Schüler mit systematischen Nullenfehlern entfielen knapp 80% aller Nullenfehler.

Diskussion der Ergebnisse

Vergleicht man die Ergebnisse der Sekundarstufe mit denen der Grundschule (3. Schuljahr), so fällt auf, dass sich die Rechensicherheit nicht erhöht hat: der Lösungsprozentsatz scheint vom 3. Schuljahr an nahezu konstant zu bleiben. Immerhin ist während der gesamten Schulzeit in der Sekundarstufe vor allem in der Hauptschule doch eine gewisse Verbesserung festzustellen, wobei die Zahlen fürs 9./10. Schuljahr wegen der geringen Anzahl der untersuchten Schüler nur mit Vorsicht zu deuten sind.

Mit ein Grund dafür, dass während der Sekundarstufe offensichtlich kaum eine Verbesserung gegenüber der Grundschule eintritt, könnte das Fehlen von gezielten Übungen und gezielten Diagnosen sein, durch die der Schüler individuell auf die bei ihm fehlerhaften Kombinationen aufmerksam gemacht wird. Immerhin zeigte sich bei einigen Schülern, dass sie in beiden Richtungen den gleichen Fehler machten (z.B. 8·7=54 und 7·8=54). Eine Nachfolgeuntersuchung in den gleichen Klassen müsste hier Klarheit schaffen, ob es sich dabei nur um sporadische Fehler oder um systematisch falsch eingeprägte Kombinationen handelt.

Die Beobactung, dass sich die Rechensicherheit nach dem 6. Schuljahr jedenfalls nicht verschlechtert hat, ist im Zusammenhang mit der Tatsache, dass ab dem 7. Schuljahr der Taschenrechner als Hilfsmittel im Mathematikunterricht zugelassen war, von besonderem Interesse. Entgegen einer weitverbreiteten Befürchtung, die Schüler würden bei Gebrauch des Taschenrechners plötzlich ihre Kopfrechenkenntnisse vergessen, deuten die gewonnenen Daten eher in die entgegengesetzte Richtung.

Wie die am Anfang dieses Beitrags angestellten Modellrechnungen zur Rechensicherheit gezeigt haben, ist ein durchschnittlicher Lösungsprozentsatz von 95% bei Einmaleinskombinationen keineswegs genügend, um z.B. einigermassen erfolgreich schriftlich multiplizieren zu können. Würde man dafür einen höchstens halb so hohen Fehlerprozensatz fordern, so zeigt sich, dass nur gut die Hälfte der Schüler unter dieser Marke liegt; für die andere Hälfte genügen die vorhandenen Kenntnisse nicht, um ihnen bei komplexeren Rechnungen ein Mindestmass an Erfolg zu garantieren.

Die starke Fehlerkonzentration auf einzelne Schüler zeigt, dass es sich in jeder Klasse lohnen würde, die Schüler, die besondere Schwierigkeiten haben, herauszufinden und ihre Fehler zu diagnostizieren. Da jeweils nur eine überschaubare Anzahl von Kombinationen speziell geübt werden müsste, könnte hier ein individualisiertes Übungsprogramm für die einzelnen Schüler zu deutlichen Verbesserungen führen. Interessante methodische Vorschläge, wie ein solches Übungsprogramm zur Automatisierung aussehen könnte, hat E. Ferner (1982) mit seinem "1-Minuten-Rechnen" entwickelt.

Während eine Automatisierung bei den hohen Kombinationen bei vielen Schülern bis zum Ende der Schulzeit nicht erfolgt, ist insgesamt im Vergleich zur Grundschule eine deutliche Verringerung des Zeitbedarfs und damit auch eine zunehmende Automatisierung zu erkennen. Dass die Schüler in der Realschule deutlich schneller als die Schüler der Hauptschule im entsprechenden Alter sind, könnte damit zusammenhängen, dass diese Fähigkeit mit bei der Auslese für die Realschule nach dem 4. Schuljahr eine Rolle spielte.

Die geringe Korrelation zwischen Fehlerzahl und Zeitbedarf hat wichtige pädagogische Konsequenzen für die Organisation von Klassenarbeiten (Tests) in Mathematikunterricht. Wenn bei einer Klassenarbeit das mathematische Wissen eines Schülers erfasst werden soll, dürfte sie nicht unter Zeitdruck geschrieben werden. Zum einen weiss man, dass unter Stressbedingungen gerade die flexiblen Denkfähigkeiten blockiert werden. Zum andern zeigt die geringe Korrelation zwischen Routinekenntnissen und Schnelligkeit: Wenn Schüler wegen Zeitbegrenzung nicht fertig werden, muss man davon ausgehen, dass häufig das Wissen vorhanden ist und nur die Zeit fehlte.

Eines der auffallendsten Ergebnisse ist das riesige Ausmass der Nullenfehler. Hier scheint offensichtlich ein blinder didaktischer Fleck zu liegen. Während bei gezielten Übungen zur Multiplikation mit Null die Fehlerquote drastisch gesenkt werden kann, wie die Ergebnisse im 3. Schuljahr gezeigt haben, scheinen beim Erlernen, Üben und Anwenden die Nullkombinationen im Mathematikunterricht mit diesen Schülern geradeswegs vergessen worden zu sein. Die grosse Zahl der Nullenfehler muss vor allem auch dann beunruhigend sein, wenn man bedenkt, dass diese Schüler grösstenteils von Anfang an moderne Mathematik gelernt haben - d.h. eine Mathematik, die sie nach den Versprechungen der Anfangszeit in besonderer Weise befähigen sollte, Einsichten ins Zahlsystem, in die Stellenwerte und damit vor allem auch in die Rolle der Null zu gewinnen.

Untersucht man die sonstigen Schwierigkeiten bei den einzelnen Einmaleinskombinationen, so fällt auf, dass die 8-er Reihe sowie die Multiplikation mit 8 zum schwierigsten gehört. Ein gezieltes Übungsprogramm zu den schwierigen 1x1-Kombinationen müsste neben den 16 hohen Kombinationen (evt. unter Verzicht auf 6x6 und 9x9) zusätzlich auch die beiden stark fehleranfälligen Kombinationen 8x4 und 4x8 enthalten.

Insgesamt lässt sich wieder eine ähnliche Schwierigkeitsabstufung wie im 3. Schuljahr beobachten: der Fehlerprozentsatz verdoppelt sich jeweils ungefähr von den Kombinationen mit 1, 2, 10, über die niederen und die gemischten zu den hohen Kombinationen.

4. Schluss

Die Untersuchung der Einmaleinskenntnisse hat vor allem ein erschreckend hohes Ausmass an Fehlern bei der Multiplikation mit Null enthüllt. Hier scheint eine grosse unentdeckte didaktische Lücke im Mathematikunterricht zu bestehen. Soll nicht bei einigen Schülern ein Misserfolg bei schriftlichen Rechenverfahren, im Bruchrechnen und in der Algebra vorprogrammiert werden, so müsste man vordringlich im Mathematikunterricht versuchen zu diagnostizieren, welche Schüler Schwierigkeiten mit der Null haben, um ihnen gezielt das Prinzip der Multiplikation mit Null erklären zu können.

Dieses Vorgehen hätte grosse Chancen auf Erfolg. Einmal konzentrieren sich die Nullenfehler auf wenige Schüler. Wenn es gelänge, die durchschnittlich 5 Schüler pro Klasse ausfindig zu machen, die systematisch auf einer oder auf beiden Seiten falsch mit der Null multiplizieren, könnte man den Fehlerprozentsatz beim 1x1 auf nahezu die Hälfte verringern.

Zum andern hat man hier gegenüber den hohen Kombinationen, die je einzeln zu lernen sind, den Vorteil, dass der Schüler nur ein Prinzip und ein einziges Ergebnis bei der Multiplikation mit Null zu lernen hat.

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ÜBER MATEMATISCHEN ANFANGSBEREITSCHAFTEN DER LEHRERSTUDENTEN

Erkki Pehkonen

1. Einleitung

Im folgenden betrachten wir die mathematische Anfangsdiagnose in der finnischen Klassenlehrerausbildung. (Der Klassenlehrer wird in Finnland in den Klassen 1.-6. Unterrich geben.) Zuerst werden die Gedanken erklärt, die hinter dem Benutzen der Anfansdiagnose liegen, sowie einige frühere Versuche und Betrachtungen. Dann werden die Minimumanfangskenntnisse hergeleitet und dargestellt. Letztlich werden kurz die Ergebnisse der letzt ausgeführten Anfangsdiagnose angegeben und kurz die der Anfangsdiagnose folgenden Massnahmen erklärt. Im Anhang ist der Anfangsniveautest.

2. Hintergrund

Im Mathematikunterricht der Klassenlehrerausbildung sind eine praktische Schwierigkeit die verschiedenen mathematischen Fertigkeiten der Lehrerstundenten. Sie haben zwar alle das Gymnasium besucht, der schulmathematische Hintergrund kann aber von Schülern des langen Mathematikkursus (und einige haben die Reifeprüfung der Mathematik sogar ausgezeichnet gemacht) zu denen variieren, die den kurzen Mathematikkursus ausgeführt und nicht Mathematik als Fach in der Reifeprüfung gehabt haben. Es kommen manchmal auch solche Studenten vor, die in der gymnasialen Oberstufe nur an dem mathematischen

Minikursus (Stundenzahl weniger als fünf pro zwei Wochen) teilgenommen haben. Die Ursache für diese Buntheit liegt an der vom Kultusministerium bestätigten Studentenwahlmethoden, die keineswegs die Studenten an Hand der mathematischen Fertigkeiten ausscheiden, sondern lieber in der Rictung der sprachlichen und sozialen Bereitschaften.

Ingolge der kumulativen Struktur der Mathematik ist die Beherrschung bestimmter Grundfertigkeiten eine notwendige Voraussetzung, um Neues zu lernen. Das Mathematiklernen dürfte sich nicht auf Gedächtnis stützen, sondern die neue Information sollte an die früher Erlernte befestigt werden. Deshalb sind das Klären der mathematischen Grundbereitschaften und das Ebnen der Differenzen eine notwendige Voraussetzung für den Erfolg des Unterrichts.

Oft hört man die Behauptung, dass der Klassenlehrer nicht viel Mathematikstudien benötigt, weil "in der Primarstufe nur die Grundrechenarten gelehrt werden". Dies stimmt aber nicht mehr heute; dazu ist es zu bemerken, dass das erstrangige Ziel der Klassenlehrerausbildung folgend ist:

> "... der Lehrerstudent ist auch in mangelhaften Umständen fähig, die verschiedenen Fächer des Curriculums für die Unterstufe der obligatorischen Schule zu unterrichten..." (Anon. 1978, 8).

Dies kann man nur durch "Überlernen" erreichen, d.h. der Lehrerstudent soll das Fach umfangreicher und gründlicher beherrschen als was die zukünftige Arbeit des Klassenlehrers voraussetzen würde.

Zur Basis der Lehrperiodenanalyse nimmt man oft das GLASERsche Unterrichtsmodell, wobei das Anfangsniveau der Schüler zwischen den Zielen und Unterrichtsordnungen hingestellt geworden ist. Eine wesentliche Voraussetzung für den Erfolg des Unterrichts ist das Kennen der Anfangsbereitschaften der Schüler.

Die Bedeutung des Anfangsniveaus kommt auch bei den Bloomschen mastery learning-Untersuchungen vor, nach denen das kognitive Anfangsniveau etwa eine Hälfte und das affektive Anfangsniveau etwa ein Viertel von den Lernergebnissen erklärt.

Im Institut für Lehrerausbildung an der Universität Helsinki hat man das Anfangsniveautesten ab Herbst 1979 für die jeweils beginnenden Lehrerstudenten gehalten. Die Resultate vom Herbst 1979 sind früher publiziert geworden (Pehkonen 1980).

In der Handelshochschule von Helsinki bemerkte man vor ein paar Jahren die mathematischen Vorkenntnisse der beginnenden Studenten so verschiedenartig, dass man immer beim Studieanfang eine mathematische Anfangsniveauarbeit vorlegen wollte. Mit Hilfe ihrer Resultate wird ein Teil der Studenten zum mathematischen Stützunterricht gelenkt, um die Differenzen vor dem Grundkursus der Mathematik (im zweiten Semester) zu verkleinern.

In Schweden hat man seit zehn Jahren die Aufmerksamkeit auf die schwachen mathematischen Anfangsbereitschaften der Lehrerstudenten gerichtet, besonders in der Klassenlehrerausbildung. In Göteborg ist Arbeitsruppe GUME seit 1974 tätig, die die mathematischen Anfangsbereitschaften jedes beginnenden Klassenlehrerstudenten getestet hat. Torsten Lindblad hat aud den Ergebnissen der vier erledigten Testen im Jahre 1978 ein Bericht (Lindblad 1978) abgefasst, das sehr klar zeigt, wie das Niveau Jahr um Jahr sinkt.

In der Bundesrepublik Deutschland ist man an den Universitäten und Hochschulen schon lange über die Schwächung der mathematischen Vorkenntnisse bekümmert gewesen (u.a. Nägerl & Harten 1980, Säckl 1981). Auf der Tagung für Didaktik der Mathematik in Klagenfurt (März 1982) konzentrierten sogar drei Vorträge (Akad. Dir. Christmann, Dr. Haussman, Prof. Kütting) auf dieses Thema. Fast einheitlich wurde auch die Verwendung des mathematischen Einführungskursus vorgeschlagen (Christmann 1983, Kütting 1982).

3. Minimumanfangskenntnisse

Die Aufgabe des Anfangsniveautests ist die Anfangsbereitschaften der Studenten in dem Umfang zu überprüfen, was wegen der Studien nötig ist. Also hat man als Startpunkt der Zubereitung der Anfangsdiagnose die Analyse des Stoffes sowie der Ziele des Lehrkursus zu betrachten. Damit bekommt man ein klares Bild darüber, welche Bereitschaften beim Anfang des Unterrichts erfordert werden. Diese Erforderungen, die für das Anfangsniveau gestellt werden, nennen wir im folgenden die Minimumanfangskenntnisse.

Der Lehrplan für die obligatorische Schule dibt indirekt einen Startpunkt fur den Mathematikunterricht in der Klassenlehrerausbildung. Nach dem Lehrplan kann man in der Mathematik auf der Unterstufe (1.-6. Schuljahr) vier Teilgebiete auseinanderbringen (Anon. 1982, 11): Grundrechenarten, Einführung in die Algebra, Geometrie und anwendbare Mathematik. Auf der Oberstufe (7.-9. Schuljahr) ist die Mathematik in sechs Teilgebiete gegliedert (Anon. 1982, 48): Zahlenbereichen, algebraische Ausdrücke, Funktionen, Gleichungen und Ungleichungen, Geometrie sowie anwendbare Mathematik. Mehr über die finnischen Mathematiklehrpläne findet man z.B. im Artikel des Autors (Pehkonen 1984).

Im Prinzip könnte man den kurzen Mathematikkursus der gymnasialen Oberstufe zum Anfangsniveau wählen, der grösste Teil der gymnasialen Oberstufenmathematik ist aber nicht dem Primarstufenlehrer der obligatorischen Schule nötig (wie Logarithmen, Differential- und Integralrechnung). Natürlich ist es sinnvoll neben der Primarstufenmathematik solche Teile der gymnasialen Mathematik zu erfordern, in denen die Elementarmathematik in allgemeinerer Form vorkommt, damit man das auf der Seite 2 erwähnte "Überlernen" erreicht. Zum Beispiel enthält das Rechnen mit den Polynomen (und mit den rationalen Ausdrücken) als Spezialfall das Bruchrechnen. Die Statistik und die Wahrscheinlichkeitsrechnung könnten auch dabei sein, weil

das Ausbildungsprogramm für den Klassenlehrer viel erziehungswissenschaftlichen Methodenunterricht enthält.

Das Erforderungsniveau des Tests und der Schwierigkeitsgrad der Aufgaben sind natürlich miteinanderverbunden. Weil ein diagnostischer Test auch auf die Einstellungen der Studenten wirkt, könnte es besser sein, bei den Grundaufgaben zu bleiben. Dann hätte man die Möglichkeit, ein hohes Erforderungsniveau für den Test zu setzen.

Der Autor hat die eben erwähnten Sachen berücksichtigend die Minimumanfangskenntnisse für den Mathematikunterricht in der Klassenlehrerausbildung zusammengestellt. Der Lehrerstudent soll mit dem Erforderungsniveau von 80 % den folgenden Stoff beherrschen:

1. Zahlenmengen (Grundrechenarten mit verschiedenen Zahlen).
2. Potenzen (auch Quadratwurzel).
3. Polynome (auch Bruchausdrücke).
4. Funktionen (Wert des Ausdrucks, lineare Funktion, Polynomfunktion, Graphe).
5. Gleichungen (Lösen der Gleichungen und Ungleichungen des 1. und 2. Grads, Lösen der linearen Gleichungs- und Ungleichungspaare).
6. Geometrie (Plani- und Stereometrie, Vektorgeometrie).
7. Anwendbare Mathematik (Verhältnis, Vergleichung, Prozent- und Zinsrechnen, Grössenverwandlungen).
8. Statistik (Mittelwert, Standardabweichung, klassische Wahrscheinlichkeit, Normalverteilung).

4. Anfangsniveautest vom Herbst 1982

Seit Herbst 1979 untersucht man im Institut für Lehrerausbildung an der Universität Helsinki, wie die Studienanfänger die wichtigsten Teilgebiete der Schulmathematik beherrschen, wenn man die eben dargestellten Minimumanfangskenntnisse (S. 4) als Kriterien benutzt hat.

Die Aufgaben des Tests hat man so zusammengestellt, dass sie die Minimumanfangskenntnisse gleichmässig decken. Aus jedem Teilgebiet hat man drei Aufgaben verfertigt; also im Test gibt es insgesamt 24 Aufgaben.

Der Test ist sog. multiple-choice-test: bei jeder Aufgabe gibt es vier Alternativen sowie die fünfte "keine der früheren". Die Wahrscheinlichkeit, dass man durch Erraten das Erforderungsniveau von 80 % erreicht, ist kleiner als eins zu Hundertmillion. Die Testaufgaben sind mechanische Grundaufgaben (siehe Anhang).

Zwei Wochen vor dem Semesteranfang wurden die Minimumanfangskenntnisse den Lehrerstudenten (S. 4) zugesendet; als Anlage waren dabei Musteraufgaben (vier aus jedem Teilgebiet) mit den Antworten sowie das Datum des Anfangsniveautests (in der zweiten Woche des ersten Semesters).

Für das Ausführen des Tests hatte man Zeit anderthalb Stunden. Die ersten Studenten waren aber fertig schon nach einer halben Stunde und der grösste Teil innerhalb einer Stunde. Beim Testen waren 61 Lehrerstudenten anwesend von den 80 zur Klassenlehrerstudien Eingenommenen.

5. Ergebnisse des Anfangsniveautests

Jede Aufgabe wurde dikotomisch ausgewertet: richtig - falsch (oder 1 - 0), damit die Maximumpunktzahl 24 war. Die Punktverteilung ist in Tabelle 1 ausgeführt.

Tabelle 1. Die Punktverteilung des Tests

Punktzahl	24	23	22	21	20	19	18	17	16	15	14	13
Häufigkeit	1	4	9	15	5	6	7	6	-	1	1	3

Punktzahl	12	11	10	9	8	7	6	5	4	3	2	1	0
Häufigkeit	2	-	1	-	-	-	-	-	-	-	-	-	-

Der Mittelwert der Punktzahlen war 19.2 Punkte und die Standardabweichung 3.1. Wenn das vorhergesetzte Erforderungsniveau von 80 % (S.97) 19 Punkte erklärt wurde, werden 40 aus den 61 Ausführungen (oder 66 %) genehmigt und 21 Ausführungen (oder 34 %) zurückgewiesen.

In Tabelle 2 sind die Lösungsprozentzahlen je nach der Aufgabe.

Tabelle 2. Die Lösungsprozentzahlen der einzelnen Aufgaben

Aufgabe	1	2	3	4	5	6	7	8	9	10	11	12
Lösungsprozentzahl	97	92	77	92	89	92	92	51	97	95	97	66

Aufgabe	13	14	15	16	17	18	19	20	21	22	23	24
Lösungsprozentzahl	97	93	98	93	89	74	85	25	56	66	61	54

Die allerschwächste Ausführung war bei Aufgabe 20 (nur 25 %), die sich an eine einfache planimetrische Situation anschliesst: das Rechnen des Umfangs eines Kreisviertels. Die zweitschwächsten

Aufgaben 8, 24 und 21 konnte man schon wesentlich besser als die eben erwähnte Aufgabe lösen (die Lösungsprozentzahlen 51 %, 54 % und 56 %). Aus diesen drei war die letzte (Aufgabe 21) geometrisch: das Rechnen des Gesamtflächeninhalt eines Quaders. Die geometrischen Aufgaben kann man aber bekanntlich schwach lösen! In Aufgabe 24 benötigte man die Elemente der klassischen Wahrscheinlichkeit; die Wahrscheinlichkeitsrechnung findet man jedoch allgemein schwierig. Dagegen war Aufgabe 8 eine Routineaufgabe der Potenzenasudrücke, welche schon auf der Oberstufe (7.-9. Schuljahr) der ovligatorischen Schule sehr viel geübt werden; deshalb könnte man das Lösen solcher Aufgaben erwarten.

Die folgenden in der Schwierigkeitsordnung waren die Aufgaben 23, 22 und 12, deren Lösungsprozentzahlen schon über 60 % waren (genauer 61 %, 66 % und 66 %). Die zwei ersten gehören zur Statistik; in der finnischen Schule wird die Statistik sehr leicht abseitsgeblieben, weil die anderen Teilgebiete der Mathematik wegen des Zeitmangels betont werden. Die letztgenannte Aufgabe behandelte die Vereinfachung des rationalen Ausdrucks, die man allgemein schon in der Schule als eine anspruchsvolle Aufgabe findet.

Jetzt betrachtet man die Testergebnisse nach den in den Minimumanfangskenntnissen dargestellten Teilgebieten. In Tabelle 3 sieht man, wie die verschiedenen Teilgebiete gelöst wurden.

Tabelle 3. Die Lösungsprozentzahlen der verschiedenen Teilgebiete des Tests.

Teilgebiete (Aufgaben)	Lösungs-prozentzahlen
1. Zahlenmengen (1,2,3)	89
2. Potenzen (7,8,9)	80
3. Polynome (10,11,12)	86
4. Funktionen (16,17,18)	85
5. Gleichungen (13,14,15)	96
6. Geometrie (19,20,21)	55
7. Anwend. Math. (4,5,6)	91
8. Statistik (22,23,24)	60
Alle	80

Die schwächste Ausführung war in der Geometrie (55 %). Hier sieht man die allgemein bekannte Tatsache, dass die geometrischen Fertigkeiten schwächer geworden sind. Fast ebenso schwach war die Lösungsprozentzahl in der Statistik (60 %). Auf diesem Gebiet war eine niedrige Leistung zu erwarten, denn die Statistik und die Wahrscheinlichkeitsrechnung kommen zum Unterricht erst in der letzten Klasse der gymnasialen Oberstufe und die Stundenzahl ist sehr gering. (Die Situation des Statistikunterrichts wird neben der Reform der gymnasialen Oberstufe (siehe Pehkonen 1984) ab 1984 besser.)

Die Lösungsprozentzahlen in anderen Teilgebieten sind wenigstens 80 % - in der Gleichungen sogar 96 %. Solche hohe Leistungen kann man aber von den Studenten erwarten, die eine Gelegenheit gehabt haben, sich für den Test vorzubereiten, wenn man dazu den Schwierigkeitsgrad und das Antwortenverfahren (multiple choice) berücksichtigt.

Zum SChluss sind in Tabelle 4 die Lösungsprozentzahlen nach den für die Aufgaben velangten Vorkenntnissen schulstufenweise verglichen geworden. Dabei kommen zwei Gruppen in die Frage: Die Aufgaben, die mittels der lauter Unterstufenmathematikkenntnisse (1.-6. Schuljahr) gelöst worden sind; solche Aufgaben sind 1, 2, 3, 4, 5 und 6. Die Aufgaben, in welchen man höchstens die Mathematik der obligatorischen Schule (1.-9. Schuljahr) benötigt, die Aufgaben 18, 22, 23 und 24 gehören nicht zu dieser Gruppe.

Tabelle 4. Die Lösungsprozentzahlen des Tests schulstufenweise

Aufgabengruppe	Lösungsprozentzahl
Nur die Unterstufenmathematik (1-6)	90
Mathematik der obligatorischen Schule (1-17, 19-21)	84
Alle	80

6. Massnahmen

Die Bedeutung des Anfangsniveautestens ist, die mathematischen Angangsbereitschaften der Lehrerstudenten zu ermitteln, damit man an der möglicht frühen Pfase den Studenten helfen könnte. Das Ziel ist, die grössten Lücken im Kenntnisse der Grundmathematik vor dem Beginn des Grundkursus (am Zweiten Semester) auszubessern.

Den Studenten, die in Anfangsniveautest nicht genehmigt sind, wird am Ende des Herbstsemesters eine neue Möglichkeit veranstaltet. Für den Wiederholungstest können die Studenten allein Üben oder an einem Ergänzungskursus der Schulmathematik teil-

nehmen, wo man innerhalb 20 Stunden die wichtigsten Fragen der Minimumanfangskenntnisse behandelt. Eine zweite Wiederholungsmöglichkeit gibt es am Anfang des Frühjahrsemesters.

In den Jahren 1979-82 hat die Ablehnungsprozentzahl im Anfangsniveautest zwischen 20 - 30 % abgewechselt. Der grösste Teil der Studenten, die im ersten Test zurückgewiesen sind, haben die Minimumanfangskenntnisse schon bei der ersten Wiederholung geleistet. Die Situation sieht aber problematisch bei einigen Lehrerstudenten aus, die vielleicht zu grosse Lücken in den mathematischen Grundkenntnissen und -färtigkeiten haben; sie sind nich fähig gewesen, ihre Leistungen nach 2 - 4 jährigen Bestrebungen zu vervessern.

Weil das Institut für Lehrerausbildung die Klassenlehrerstudenten von der etwa zehnmaligen Bewerbermenge wählen kann, wäre es auch aus dem Standpunkt der Studenten besser, wenn man in den Wahlkriterien irgendwie das Beherrschen der Mathematik mitnehmen könnte. In Schweden hat Lindblad (Lindblad 1978, 59-60) vorgeschlagen, dass die Mathematiknote als notwendige Forderung in die Studentenwahlmethoden mitgenommen würde. Dasselbe könnte auch bei uns vernüftig sein.

ANFANGSDIAGNOSE Herbst 1982 Name: _____

Beringe in jeder Frage die Alternative A, B, C, D, E, die du als richtig betrachtest. Man wählt nur eine Alternative.

1. Berechne $6 \cdot 3 + 72 : 3 - 8$.
   A -34, B -9, C 9, D 43, E keine der früheren.

2. Berechne $8{,}5 \cdot 5{,}6 - (8{,}5 + 5{,}6)$.
   A 14,1, B 30,8, C 31,7, D 33,5, E keine ker früheren.

3. Berechne $\frac{5}{4} + \frac{2}{3} - \frac{1}{6}$.
   A $\frac{3}{2}$, B $\frac{7}{4}$, C $\frac{7}{12}$, D $\frac{6}{13}$, E keine der früheren.

4. Wieviel Prozent sind 32 kg von 80 kg?
   A 25, B 40, C 48, D 60, E keine der früheren.

5. 250 s ist ebenso gross wie
   A 2 h 5 min, B 2,50 h, C 2 min 50 s, D 4 min 10 s, E keine der früheren.

6. In welchem Verhältnis steht 30 p zu 40 mk?
   A 3:4, B 3:40, C 40:3, D 4:30, E keine der früheren.

7. Berechne $(-2)^4$.
   A -16, B -8, C 8, D 16, E keine der früheren.

8. Vereinfache den Ausdruck $a^4 \cdot a^n (a^2)^{-n}$.
   A a^2, B a^{2n}, C a^{n+4}, D a^{4-n}, E keine der früheren.

9. Berechne $\sqrt{100} - \sqrt{36}$.
   A -8, B -4, C +4, D +8, E keine der früheren.

10. Die Summe der Polynome $x^2 - x + 3$ und $x + 1 + x^3$ ist
    A $2x+4$, B $2x^2+4$, C $2x^3+4$, D $2x^5+4$, E keine der früheren.

11. Den Ausdruck $-2x(x - 2) + x^2$ kann man in die folgende Form vereinfachen:
    A $-x^2-2$, B $-x^2+4x$, C $-x^2-4x$, D $-x^4+4x$, E keine der früheren.

12. Vereinfache den Ausdruck $\left(\frac{1}{2x+4} - \frac{x}{x^2-4}\right) : \frac{4}{x^2-4x}$.
    A $\frac{x^2 - 2x}{8x + 16}$, B $\frac{2}{x^3 - 4x}$, C $\frac{2}{x(x-2)^2}$, D $\frac{x}{8}$, E keine der früheren.

13. Die Lösung der Gleichung $x(3x + 8) = 8(x + 6)$ ist
    A x=0, B x=6, C x=8, D x=12, E keine der früheren.

14. Die Lösung des Systems $x+y = 3$, $2x-y = 0$ ist
    A x=2, y=1, B x=1, y=-2, C x=3, y=0, D x=1, y=2, E keine der früheren.

15. Welche Ungleichung erfüllt die Zahl x=6?
A $x+1<4$, B $1-x>0$, C $x:2>1$, D $2x<8$, E keine der früheren.

16. Berechne den Wert des Ausdruckes $\frac{5x+1}{3}$, wenn x=-5 ist.
A -8, B -6, C 6, D 8, E keine der früheren.

17. Welche der folgenden Geraden läuft durch die Pünkte (0,2) und (1,0)?
A $y=-2x+1$, B $y=2x+1$, C $y=-2x+2$, D $y=2x+2$, E keine der früheren.

18. Die Summe der Koordinaten des Scheitels in der Parabel $y=x^2+2x$ ist
A -2, B -1, C 0, D 2, E keine der früheren.

19. Wenn die Katheten des rechtwinkligen Dreiecks 12 cm und 4 cm sind, ist der Flächeninhalt des Dreiecks
A 3 cm^2, B 8 cm^2, C 16 cm^2, D 48 cm^2, E keine der früheren.

20. Berechne den Umfang des Kreisviertels (mit der Genauigkeit von zwei Ziffern), wenn der Radius 2,0 m ist.
A 5,1 m, B 7,1 m, C 8,0 m, D 13 m, E keine der früheren.

21. Die Kanten des Quaders sind 2m, 3m und 4m. Berechne dessen Gesamtflächeninhalt.
A 9 m^2, B 24 m^2, C 26 m^2, D 52 m^2, E keine der früheren.

22. Die Summe der achtzig Zahlen ist 186. Berechne den Mittelwert.
A 2, B 88, C 248, D 13440, E keine der früheren.

23. Die mittlere Abweichung der fünf Zahlen ist 1,7. Aus jeder Zahl wird 1 substrahiert; berechne die mittlere Abweichung der neuen Zahlen.
A 0,588 , B 0,7 , C 1,7 , D 2,7 , E keine der früheren.

24. Die Münze wird dreimal geworfen. Wie gross ist die Wahrscheinlichkeit, genau 1 Wappen zu bekommen?
A $\frac{1}{3}$, B $\frac{2}{3}$, C $\frac{1}{4}$, D $\frac{3}{4}$, E keine der früheren.

HINTERGRUND

Abiturjahr 19__

Mathematikkursus in der gymnasialen Oberstufe: kurz / lang

Die Mathematiknote in der Schule __

Die Mathematiknote im Maturitätsexamen __

Der Test war leicht / mittelmässig / schwierig.

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AN ITEM ANALYSIS OF TESTS IN MATHEMATICS APPLYING LOGISTIC TEST MODELS

Summary of a thesis

Ritva Koponen

The aim of this thesis was to apply logistic test models to tests in Mathematics for the purpose of analyzing learning difficulties. At the same time the aim was to come to a deeper understanding of the structures of the models used and to develop the characteristics of the models from the point of view of theory. Since this is an area that has not been studied for more than the last few decades, the theory behind is still in a state of flux.

The thesis may be divided into three parts, each dealing with a different aspect of the tests. In the first part, all errors made by the pupils were analyzed, a flow chart was made of the most frequent errors. Pupils were found to make a large number of error types at the outset of tuition. This high error frequency persisted at later stages as well in the case of weaker pupils. At later stages of instruction, with more learning having occured, errors tended to decrease, both from the quantitative point of view and from the point of view of the number of error types made.

The second part concentrates on the application of the simple logistic model to the corpus. Furthermore the results were compared to results obtained by traditional test theory. Initially items were tested for fitness with the model. In this test the two test statistics T_1 and T_2, presented in the NEWRATE programme were compared with each other.

It was found that the test statistic obtained by logarithmic transformation was more feasible for its symmetricity of skewness and kurtosis. In addition, T_3, a new test statistic was developed, with characteristics quite similar to those of the test statistic T_2.

A formula was developed for the variance of the residual z^2_{vi}, a formula which clearly demonstrated the relationship between the variance and the term $\beta_v - \delta_i$. Testees' fitness for the model was examined briefly, using the same methods as for the test items. Reasons for misfit were examined separately, firstly for items and secondly for testees. The biserial correlation coefficient and the test statistic T_2 were compared with each other in order to measure the goodness of the test items.

Information provided by the item and by the test was approached by comparing Birnbaum and Fisher's concepts to the information-theoretical approach. Information functions were drawn for each test on the basis of the empirical data. Additionally a method was presented for combining the tests, once it was ascertained that both consecutive tests have a sufficient number of common items.

The third part applied the linear logistic test model to examine the structure of the items. As a result, operations difficult for the pupils were found, and which caused failure in the entire item. An estimate for each operation was calculated for the difficulty parameter with its confidence limits. The consecutive combination of tests was based on these parameters of difficulty. It was considered essential

to standardize the difficulty parameters of the items before application of the conformity test between the item difficulties of the simple logistic model and the linear logistic model respectively. Theoretical treatment was given, amongst other things, to the question under which conditions completion of the entire task was easier than the error-free completion of the individual test operations.

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